ACTIVITIES ENHANCING GIFTED CHILDREN'S CREATIVITY AND REASONING

JARMILA NOVOTNÁ
Charles University in Prague, Faculty of Education

In this paper, one type of didactical unit suitable for cultivation and motivation of mathematical talents is presented. It has been developed for investigative activities but can be easily modified for use in other mathematical domains. The situation “My lucky number” is used for illustration.

1. INTRODUCTION

The study described in the paper contributes to the research focusing on designing pedagogical methods and educational tools whose aim is to motivate potentially talented students in mathematics. We will concentrate mainly on creating interesting tasks that engage students and encourage them to develop their mathematical talents. Students should be challenged to create questions, to explore, and to develop mathematics that is new to them. They need opportunities for sharing their discoveries with others. At the same time, these activities aim also at improving the ways in which students learn mathematics. (Novotná, 2004)

The study focuses on the following issues:

- What are the properties of problems that help to develop their solvers’ creativity?
- How does one recognise whether the taught mathematical knowledge was incorporated into the students’ structure of mathematical knowledge?
- Which difficulties that students face when solving a problem can be overcome without external help and which need the teacher’s intervention?

The paper is a part of the author’s longitudinal research focusing on best practices for developing understanding of mathematics. The perspective used is the theory of didactical situations (Brousseau, 1997).
2. THE ELEMENTS OF MATHEMATICAL TALENT

Mathematical talent refers to an unusually high ability to understand mathematical ideas and to reason mathematically, rather than just a high ability to do arithmetic computations or get top grades in mathematics (Miller, 1990; Stanley & Benbow, 1986). Students may show their special talents in mathematics in various ways. There exist several lists of characteristics of talented students. This paper refers to the list presented in Makrides (2006), paying attention mainly to the following selection of characteristics connected with the topic of this paper:

- An unusually keen awareness of and intense curiosity about mathematics.
- A high ability to think and work abstractly and the ability to see mathematical patterns and relationships.
- An unusual ability to think and work with mathematical problems in flexible, creative ways rather than in a stereotypic fashion.
- An unusual ability to transfer learning to new, untaught mathematical situations.

Mathematical activities for (not only) gifted students should stress mathematical reasoning and develop independent exploratory skills (Niederer & Irwin, 2001). This can be realized by using problem solving activities and discovery learning, looking for patterns etc.

3. A-DIDACTICAL SITUATION

The teaching/learning process can be characterized as a sequence of situations (natural or didactical) that result in modifications in the students’ behaviour that are typical of getting new knowledge (Brousseau, 1997). Privileged to this process are the so called a-didactical situations where the teacher passes some of the responsibilities for the learning process onto his/her pupils. On the teacher’s part, it actually means delegating power, for the students it means gaining control; we speak about the process of devolution, where the teacher manages in a didactical situation to put the student in the position of being a simple actor in an a-didactical situation. The pupils themselves, without the teacher’s direct intervention, are investigating and discovering, they are creating a model and checking its correctness and usefulness, or they are creating a different model that they consider more useful etc. Their activity is controlled only by the learning environment and their knowledge, not by the didactical activity of the teacher. Each pupil becomes responsible for getting the required results. The teacher’s task is to both facilitate such situations and institutionalize the information obtained by the pupils. The knowledge is further utilized and developed with the teacher’s help.
And it is the use of a-didactical situations that is an effective instructional technique for gifted students that promotes self-initiated and self-directed learning. In the Theory of Didactical Situations (Brousseau, 1997), a-didactical situations have three phases:

- **Situation of action** – the learner decides and acts on the milieu; it is of no importance whether he/she can or cannot identify, make explicit or explain the necessary knowledge.
- **Situation of formulation** – at least two actors are put into relationship with the milieu; their common success requires the formulation of the knowledge in question.
- **Situation of validation** – a situation whose solution requires that the actors establish together the validity of the characteristic knowledge of this situation; its effective realization thus depends on the capacity of the protagonists to establish this validity explicitly together).

**Institutionalization** is a situation which reveals itself by the passage of a piece of knowledge from its role as a means of resolving a situation of action, formulation or validation to the new role of reference for future personal or collective uses.

### 4. DIDACTICAL SITUATION: **MY LUCKY NUMBER**

New educational materials put the stress on experimentation, data collection, observation, rule discovery, generalization, and hypotheses testing. Taking into account diversity of learning styles, such teaching strategies also promote an individual approach towards the process of education. (Favilli, 2006)

In the following text, a didactical situation is presented and analyzed from the perspective of developing the students’ ability to investigate in mathematics and to make use of the found properties of mathematical objects for further discoveries. The situation offers the opportunity for independent discovery of a successful problem-solving strategy.

#### 4.1. **My Lucky Number: not only for gifted students**

The didactical proposal *My lucky number* presented here is a contribution to the LOSSTT-IN-MATH project (Favilli, 2006). The basic “open situation” is formulated as follows:

Choose a number. Square each of its digits and add the squares to get a second number. Square the digits of the second number and add the squares to get the third
number. Continue this way to get a sequence. If the sequence reaches 1, the original number is called lucky. If not, it is called unlucky. (Bastow et al.)

The main mathematical topics for development are arithmetic and application of algorithms. The main aims for learners are making problem solving more effective by discovering regularities and presenting results clearly.

The a-didactical situation was organized following the phases as described above:

- **Situation of action**: The goal of this phase is to let students tackle the problem and try to discover the first properties of the created sequences. Children are working in pairs or threes. The teacher expresses clearly that they are allowed to use any solving strategy they want. Besides this, no hint is given by the teacher. The work in pairs and threes enables students to start formulating some ideas already in this phase.

- **Situation of formulation and validation**. Speakers of all groups present to the whole class their discoveries. In order to be understood, groups have to find a clear way of describing their problem-solving strategies. They might use any means to express clear ideas, including any type of models (e.g. graphical or symbolic representations). At the same time, they are obliged to compare their strategies of discovering with these already presented by other groups in order to recognise the differences or congruities in them. In case that an incorrect discovery, problem-solving strategy or justification is presented, the task of the class is to persuade the presenting group in a suitable way about the mistakes in their solution.

The a-didactical situation provides opportunities to employ various thinking processes. Progress in them can be achieved without a high level of mathematical content; students of all abilities are expected to be able to make some progress. (Bastow et al.)

### 4.2. Extensions for gifted learners

The most frequent discoveries uncover the answers to simple questions like

- Which numbers are lucky, which are unlucky?
- Is there a sequence that could help to complete another sequence?
- Try to draw a diagram representing possible forms of sequences.

Gifted learners can continue beyond the above discoveries. They can deal with questions like

- Can you predict in advance any numbers that will be lucky/unlucky?
- Is there a largest lucky number?
- For unlucky numbers, is the cycle always the same?
- What are the chances that a number is lucky/unlucky?
Are lucky numbers more often odd or even?
What proportion of the numbers from 1 to 100 is lucky/unlucky?
The extension could be proposed e.g. by adding cubes of digits instead of squares.

4.3. Relationship of the activity to other school mathematics domains

As mentioned above, the original mathematical topic of the activity is arithmetic. Several findings are based on the properties of the used operation. Students learn for example about the consequences of commutativity of the operation on mathematical reasoning. The activity *My lucky number* can serve as the starting point to study Pythagoras’ triples of numbers.

The activity may be extended to *algebra* by looking for general formulations of proposed properties, trying to find proofs etc. This aspect can be supported by the teacher posing appropriate questions, some of them already presented in the previous paragraph. The relation to *geometry* is less obvious. But there is no doubt that it can be related to the properties of right triangles, especially the Pythagorean Theorem. The relation to basic facts of probability and statistics can be built by the ‘statistical elaboration’ answering for example the question: What is the probability that a number between 1 and 50 is lucky/unlucky?

For gifted students, the reasoning competence can be enhanced by this activity. For example the role of *working backwards* can be discovered during situations proposed to learners.

The emphasis in the proposed situation is more on the process than content. At the same time, as far as gifted students are concerned, the attention should not only be paid to the way the student(s) undertake the activity but also to completeness of the obtained results. The open-endedness of the basic assignment enables the existence of several correct solutions instead of the only unique solution of standard school problems.

4.4. Experience from classrooms

The activity *My lucky number* was included in mathematics lessons in different forms: as a project solved by the students at home with subsequent presentation of discoveries in the classroom, as an activity solved in the classroom under the management of the teacher or as the above described a-didactical situation. In all cases, the results confirmed that the activity held the interest of all the participants. Its investigative nature naturally led the students to self-correction of their misunderstandings and misconceptions. It offered using students’ informal strategies that go beyond the frame of their previous experience and knowledge acquired during the school mathematics lessons. Several mathematical
competences were developed, e.g. mathematical thinking competence, reasoning competence, and communicating competence. (Favilli, 2006) Students evaluated by their teachers as gifted in mathematics usually looked for regularities that would shorten the work, whereas the others preferred more time consuming investigation of all possibilities.

5. CONCLUDING REMARKS

The activity My Lucky Number has shown at least some of the paths to preparation of a simple mathematical situation for developing gifted students’ creativity. (Novotná, 2005) If the students could use mathematics on relevant problems more often, the engagement and simultaneously the ability to understand and use mathematics would increase. With this also the possibility that they, even after having finished school, could use mathematics in new situations would increase.

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PROBLEM POSING STRATEGIES OF MATHEMATICALLY GIFTED STUDENTS

ILDIKÓ PELCZER,
FERNANDO GAMBOA RODRIGUEZ
CCADET, Universidad Nacional Autonoma de Mexico, Mexico DF

Abstract.
In the paper we present a preliminary analysis of mathematically gifted students’ problem posing strategies. 21 pupils, members of the Mexican extended Olympic team, completed a free context problem posing task in the domain of numerical sequences. The task consisted of proposing three sequence problems such to have an easy, one of average difficulty and a difficult problem. After completing the task, the students answered a questionnaire about the process they followed. Here we synthesize their answers along several dimensions of the problem posing process and show how these results fit into a dual-process view of creativity.

INTRODUCTION
The term problem posing has been used to designate different situations. For mathematicians problem posing refers to the process by which they obtain a problem that has not been solved yet by anyone. In most studies, though, problem posing means the formulation of novel problems with the solution unknown at least for its creator (Van den Heuval-Panhuizen et al. 1995). In other contexts it is understood as reformulation of an existing problem (Cohen & Stover, 1981). Besides defining what problem posing is, we need to identify the circumstances in which the process occurs. Stoyanova (1998) identified three categories of problem posing experiences: a) free, b) semi-structured and c) structured problem posing situations. In free situations students have no restriction about the problem posing task; in semi-structured situations students are confronted with open ended problems, meanwhile in structured situations students reformulate an existing problem.
By taking into consideration that problem posing is such a multifaceted construct we can ask how it is possible to analyze it. There are at least two ways to do that: first, we can look at the generated problems and analyze them from different points of view and, second, to analyze the processes involved in problem posing. Silver and Cai (1996) defined a multiple-step data coding scheme in order to analyze the arithmetical problems posed by middle school students. This problem-focused analysis examined the problems in several phases: verifying first the nature of the posed question (non-mathematical, mathematical or statement), then classified them as solvable and non-solvable. The last phase of the categorization consisted from a semantic and linguistic analyses of the problems. Another way of looking at problem posing is by focusing on the underlying processes. Christou et al. (2005) proposed a theoretical model of problem posing as consisting from several processes. They propose four processes: editing quantitative information, their meanings and relationships; selecting quantitative information; comprehension and organization of quantitative information and translating quantitative information from one form to another.

Our approach differs from the two mentioned one. We make a qualitative analysis of the processes involved in problem posing, trying also to identify the key-elements students’ use as reference points. Therefore, we are interested in identifying from where students start when they engage in a problem posing task, how they monitor their own progress, what criteria they establish in order to make this monitoring viable and in which ways their knowledge and experience influence the whole process. Our results argue in favor of the view that problem posing is a creative process and fits into a dual-state model of creative cognition (as proposed by Finke et al., 1992; Liu, 1998, for example).

**METHOD**

21 students, members of the extended Mexican mathematical Olympic team, participated in the experiment. These students have a considerable experience in problem solving, participate in preparatory courses and are habituated dealing with difficult, math Olympiad style problems. The participation in preparatory courses gives these students a knowledge base that goes beyond the high school mathematics and consists not only from problem solving techniques, but also of criteria, theorems and advanced mathematical concepts.

Students were asked to pose three sequence problems such as to have an easy, an average difficulty and a difficult problem. They disposed of two hours to respond. Once they finished this task, they had to answer to a questionnaire regarding aspects of their problem posing process. The questions were about the following aspects of the problem posing process: the existence of an initial idea (for each
problem of different difficulty), the change during generation of this idea, problem types from which to start the generation process, a theorem or generalization as from where to trigger the problem posing process and the difficulty criteria they used. The questionnaire was important for us, since we were also interested in an analysis of the process of generation students followed.

ANALYSIS OF THE PROBLEMS

First of all, we need to separate the problems from exercises and then identify well/formulated problems. A problem was considered to be well formulated if all necessary elements for its analysis were specified correctly. A problem is solvable if it has a solution within the current formulation. We made another type of analysis based on the correctness of the problem. If the claim was true, then the problem is considered as correct, incorrect otherwise. As far the problem formulation is concerned, we identified textual and formal descriptions.

Table 1. Preliminary categorization of the posed problems

<table>
<thead>
<tr>
<th>Problem categorization</th>
<th>Easy</th>
<th>Average</th>
<th>Difficult</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exercise vs. Problem</td>
<td>3 vs. 18</td>
<td>0 vs. 20</td>
<td>20</td>
</tr>
<tr>
<td>Textual vs. Formal</td>
<td>2 vs. 19</td>
<td>1 vs. 19</td>
<td>1 vs. 19</td>
</tr>
<tr>
<td>Well-stated vs. Not well-stated</td>
<td>21 vs. 0</td>
<td>17 vs. 3</td>
<td>16 vs. 4</td>
</tr>
<tr>
<td>Solvable vs. Not-solvable</td>
<td>21 vs. 0</td>
<td>17 vs. 0</td>
<td>11 vs. 5</td>
</tr>
<tr>
<td>Correct claim vs. incorrect</td>
<td>19 vs. 2</td>
<td>17 vs. 0</td>
<td>9 vs. 2</td>
</tr>
</tbody>
</table>

It can be seen from the table that students perform better in case of easy problems. In case of difficult problems they try to pose a problem such to involve some complex knowledge, but they do not necessarily know to handle it well, so the number of incorrectly stated problems is higher than in other cases. On other hand, more difficult problems are not solvable.

Borasi (1986) defines the term *problem formulation* as the explicit definition of the task to be performed. We identified four formulation types between the posed problems (table 2).

Table 2. Number of problems in different formulations

<table>
<thead>
<tr>
<th>Problem difficulty &amp; formulation</th>
<th>Easy</th>
<th>Average</th>
<th>Difficult</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prove that property P</td>
<td>12</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>Analyze property P</td>
<td>3</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>Construct such to have property P</td>
<td>-</td>
<td>-</td>
<td>2</td>
</tr>
<tr>
<td>Compute the value of property P</td>
<td>4</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>
As it can be seen in table 2, the problems that ask for computing a term or some characteristic about the sequence are few, meanwhile there are more problems of the type analyze. Students became more careful in case of the difficult problems: not being always sure that the problem can be solved they prefer to let these problems open ended, giving a formulation of the style: Analyze if there is any sequence with the property \( P \). The same explication goes for problems formulated as Prove. In case of easy problems these are easily verifiable properties (mostly by an inductive reasoning).

The subjects of the formulations refer mostly to divisibility, sum of terms, inequality relations between terms and sum of terms, the form of the general terms in case of easy problems. In average difficulty problems questions' subject are about identifying the membership of a term to the sequence, proving that a sequence has no term of a specific form or that a sequence has infinity of terms of a specific form. In difficult problems questions are about divisibility, prime number terms, perfect square terms, and integer number terms. The subject of these questions differ of the usual sequence problems questions in sense that there are almost no questions about convergence, limit computing, monotony, study of bounds, etc., questions typical to textbook sequence problems. Only few students made up problems that reassemble textbook problems and their tendency could be seen over the three problems they proposed and the knowledge they referred to when posing those problems. Our result, in this sense, is similar to the one obtained by Ellerton (1986) who appeals to the experience of the child in order to explain the types of problems he can make up. With very few exceptions, Olympic students try to make problems as they have seen at competitions.

As far the specifications given in the problem are concerned, they can be synthesized as follows. In easy problems, the sequence usually is specified by an explicit formula for the general term that is computable. Algorithmic knowledge is usually enough to treat these problems. In case of problems of average difficulty, even when there is an explicit relation between the terms, the general term is not always computable (in hard problems, too). However, the most important distinctive feature of average and hard problems is the problem’s question. In order to answer it, one needs more than the usual knowledge related to sequences or reasoning skills – mostly knowledge from number theory.

**ANALYSIS OF THE PROCESS**

By this analysis we mean to synthesize student's answers regarding their start point and ideas involved in the task execution and their criteria of difficulty.

We asked the students if they had an initial idea before starting the problem posing process. Then, another question was if this idea changed during the process and
why. In total we had 7 general answers at initial idea (that is, it was not specified at which problem would apply the description) and 3 at each problem type. What students reported as initial idea can be interpreted as a very general view about what would involve the problem, what would be about or what kind of knowledge would be necessary for it. Although this is a very shallow view of the problem it guides from the beginning the posing process: the student will try to combine ideas and in the final form of the problem will try to “hide” the original idea. However, this idea sometimes has to be changed. In the study we had 7 students who reported such case and this fact highlights an interesting point in the process: starting from an initial idea, the student gathers the knowledge and experience such to obtain the problem he wanted, however in the process he needs to make changes such to comply with the requirements of a mathematical problem (at least to be well formulated). Such a result is compliant with the dual-process view of creativity: there is a generative phase in which new material is produced (in this case: parts of the problem) followed by a revision, an analytical phase, during which it is analyzed if the new material satisfies some criteria established beforehand.

Other questions regarded the knowledge that student use in order to bootstrap the problem posing process. There were three questions related to this issue.

The first one asked the students if they thought of a particular theorem or some result from which to start. There were 7, 6 and 8 students who responded affirmative to this question for difficulty level easy, average and high. Usually, students report inductive reasoning, known inequality or identity relations in this category. Once they decide to use some theorem, the next step is to analyze how it is possible to build a problem from it and they engage in a creative process described in the previous paragraph. These theorems are known at high school level (they are usually part of the high school curricula), but it is not the knowledge that makes a difference, but the way they handle it. It is expected that students who participate in mathematical competitions to be able not only to gather knowledge significant for a task, but also to be able to transform it into an adequate form.

The second question regarded problem types. In a specific domain, students usually identify problem categories, each of them having specific characteristics. Some textbooks are organized by problem types, further facilitating in students the formation of such vision about the problems. There were 15, 14 and, respectively, 9 students who answered affirmatively to this question at easy, average and difficult problems. They described the problem type as “growing sequence” or “problems where you need to make an individual analysis of some term in order to know something about the sequence”. However, the important aspect of their definition of such “types” is in the generation process. Often, the problem type is related with the initial idea. In some terms, the problem type can be seen as a second step in the problem posing process, after the initial idea.
The third question refers to the use of generalization and was formulated as: In the problem posing process did you thought of a particular result or problem that you tried to generalize afterwards? Only one student answered affirmatively to this question. Generalization could be seen as a natural way of getting new problem of old ones, however it seems that these students prefer to use knowledge that is beyond the typical high school curriculum for getting more difficult problems.

As far difficulty criteria are concerned they can be classified into three groups. Nine students considered the complexity of the knowledge required for the solution as main criteria for establishing the problem's difficulty level. Four students appealed to how known is the knowledge required for the solution. Eight students referred to themselves, to their knowledge level and the time required generating or solving the problem in order to set the difficulty of the problem. We hypothesize that the difference between self-referencing and knowledge-referencing the difficulty comes from the difference in mathematical preparation.

CONCLUSION

In the present paper we presented an analysis of mathematically gifted student's problem posing strategies. The preliminary results show that students part from an initial idea or some reference point in the problem posing process. Then they start a “generate-evaluate” process, characteristic to creative cognition, during which they adapt their strategy such to comply with the general (regarding mathematical correctness) and self-imposed (regarding difficulty level, knowledge level) requirements. As far the quality of the proposed problems is concerned it can be said that most of the problems are beyond practice sequence problems reminding more of problems for advanced students. The knowledge involved in their generation and necessary for their solving is often far from what is typically related to sequences (like convergence criteria, monotony, etc.) and it is more related to number theory aspects (by the use of integer sequences, divisibility and prime number issues). Such an analysis of problem posing task can give an interesting view on how students handle their own knowledge and, also, how their knowledge is structured.

REFERENCES


TOWARDS PROMOTING CREATIVITY IN MATHEMATICS OF PRE-SERVICE TEACHERS: THE CASE OF CREATING A DEFINITION

ATARAH SHRIKI
Oranim Academic College of Education

Abstract
In this paper I describe some learning episodes aimed at allowing pre-service teachers to experience the process of developing their own creativity in mathematics. The experience was based on these pre-service teachers' ideas regarding the ways in which creativity of pupils might be developed. During a period of six weeks the pre-service teachers were engaged in 'inventing' geometrical concepts and examining their properties. The episodes described in this paper show that the pre-service teachers believed the experience they had undergone was valuable for increasing their awareness of the issue as well as developing their own creativity in mathematics.

INTRODUCTION

According to the NCTM Standards (2000) one of the aims of mathematics education is to enable pupils to think creatively and flexibly about mathematical concepts. However, in order to be able to develop their pupils' creativity it is essential for teachers that they hold suitable pedagogical knowledge. Since many pre-service teachers (PSTs) did not experience the essence of creativity in mathematics when they were pupils, such pedagogical knowledge should be acquired during the period of their training. Therefore, within the methods course I teach I attempt to develop the PSTs' comprehension regarding the meaning of creativity in mathematics, and enable them to experience learning within various environments that encourage and support the development of creativity.

In this paper I present few episodes taken from one possible example of such an environment, where PSTs generated new concepts and studied their properties. In the first section there is a description of the environment and some of the PSTs'
utterances. In the second section there is an examination of the process the PSTs had undergone. Finally, there are some concluding remarks.

A THREE-PHASE PROCESS OF DEVELOPING CREATIVITY IN MATHEMATICS

In this section I describe the experience of my class of 17 PSTs, while aiming at working like ‘real mathematicians’ and gaining an insight into creativity in mathematics. The experience was initiated as a result of a class discussion concerning the image of a ‘good teacher’. This experience was a three-phase process, and lasted for six weeks. At the first phase the PSTs were asked to invent a new concept, and offer a suitable name for the concept. At the second phase they presented their ideas in class, and discussed them. At the third phase they had to find some of the properties of the new defined concept, and formulate relevant theorems. During the entire process they were asked to reflect on various aspects concerning creativity. The following describes each phase in more details.

CONTEXTUAL FRAMEWORK

At the beginning of the methods course I teach, I ask my class of PSTs to describe the way they perceive the image of ‘a good teacher’. The methods course is an annual course, and is learned in their second year (out of four) of studying towards becoming middle and secondary mathematics school teachers. Subsequent to the production of a long list that includes the characteristics of ‘a good teacher’, we analyze the utterances, delving into it in an attempt to reach a ‘common language’ for the course. The same discussion is repeated at the last lesson, producing a new list, comparing it with the previous one, analyzing the differences between the two lists and discussing the learning experiences that influenced changes in perception.

In one of the classes, in the framework of the first lesson, the PSTs’ responses referred, among others, to the importance of developing pupils’ creativity. This was unusual, because usually the PSTs do not relate to creativity at the beginning of the year but only at the end, after experiencing several examples aimed at developing creativity. Focusing on ‘creativity’, the PSTs were asked first to explain how they perceived the meaning of creativity and then to suggest ways and tools aimed at developing pupils’ creativity. The excerpt that follows is a part of the discussion that took place in the class (T – teacher, PST – pre-service teacher):

[1] T: "You asserted that a ‘good teacher’ is one who concerns about developing his pupils’ creativity. What do you mean by ‘creativity’?"

[2] PST: "I think it is the ability to solve non-routine problems".
Promoting Creativity:
The Case of Creating A Definition

[3] PST₂: "Wait, I don't fully understand your question. Do you mean 'creativity' in general or just ‘creativity in mathematics’?"

[4] T: "This is a very good question. What do you think – is there any difference between them?"

[5] PST₂: "Sure! I don't think that every creative painter or musician is also a creative mathematician, and vice versa”.

[6] T: O.K. although we still didn't reach any agreement as regards to what creativity actually means, let us limit ourselves to discussing the meaning of 'creativity in mathematics'. What do you consider as ‘creativity in mathematics’?"

[7] PST₃: "Providing original proves to standard problems”.

[8] PST₄: "Raising and implementing imaginative ideas”.

[9] PST₅: "Inventing new theorems and theories”.

[10] PST₆: "To be a genius!”

(At that point several PSTs started to talk aloud, expressing their reservation from defining ‘creativity in mathematics’ as geniuses). Then one of them said:

[11] PST₇: "Geniuses is something innate. Creativity is something you can develop with a proper guidance”.

[12] PST₈: "According to my experience, there is not always a direct connection between being good in mathematics and being creative. I remember several lessons in which substitute teachers came to the class, and presented some non-routine problems. I was surprised when I realized that not always the best pupils were the first to answer. I think that because the problems did not require algorithmic thinking but rather a creative one, other pupils were capable of solving them”.

We continued the discussion, examining issues such as – Why is it important to be 'mathematically creative'? Do you consider yourself as 'mathematically creative'? How can you recognize a 'mathematically creative' pupil? How can you determine whether a certain product is an outcome of creativity? Finally I asked:

[13] T: "Considering all the interesting things you said about creativity, how can we, as teachers, promote the mathematical creativity of our pupils?”

(Few seconds of silence)

[14] PST₉: "It is very difficult to answer this question, because we didn't experience it in school, when we were pupils, or even here at college. I guess our teachers had no idea as well…”

CMEG-5 203
(Humming of approval…)

[15] PST₃: "I think we should let our pupils work like real mathematicians do".
[16] T: "What do real mathematicians do?"
[17] PST₃: "Building theories".
[18] T: "How do they build theories?"
[19] PST₁₀: "They formulate concepts, axioms, and theorems, and base their theory upon them".
[20] T: "That's interesting. Let's try to be real mathematicians. Let's try to formulate new concepts, raise conjectures, and prove or refute them".
[21] PST₁₁: "What do you mean? We are not Pythagoras…"
[22] T: "And what do you mean by that?"
[23] PST₁₁: "I don't believe pupils are the ones who have to discover new concepts, theorems or regularities. All these can be found in the books. The pupils have to acquire knowledge. Part of this process of acquiring new knowledge is expressed by the pupils' ability to prove known theorems. Not to invent or discover them".

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[24] T: "You mentioned Pythagoras, so let's think about Geometry. Can you think of a new geometrical concept, one you had never heard of before?"
[25] PST₁₁: "I believe mathematicians had already invented everything they could think of…Who are we to challenge them?"

(Laughter… Humming of disagreement…)

FIRST PHASE – INVENTING A CONCEPT

The above conversation inspired me to change the planning of the course. I continued:

[26] T: "Can you invent a new concept on the basis of previously known concepts?"

(Silence).

[27] T: "Your assignment for our next meeting is to think of a new geometrical concept, based on the concepts you are familiar with".

The PST were given a week for inventing a new concept, without any further explanations or guidance.
SECOND PHASE – PRESENTING THE CONCEPT

At the beginning of the following session I was curious to hear the PSTs' experiences during the past week. Each student exhibited the name s/he chose for the concept and its definition. The PSTs invested efforts in finding an attractive or sophisticated name for the invented concept. All the defined concepts were comprised of basic figures such as circle, triangle and quadrilateral and one typical example that satisfies each definition was attached. Some definitions were formulated in an unambiguous manner, and together we examined all the definitions, refined them, and corrected them. The PSTs were enthusiastic and they all participated actively in the discussion. Before the end of the lesson I asked the PST whether they find any connection between the assignment and the development of creativity in mathematics:

[27] PST11: "As oppose to what I said last week, I now believe pupils have the ability to invent new concepts. When you gave us the assignment I was a little frustrated, because I didn’t know where to begin. I took a textbook and examined some of the concepts in the book. I noticed that they all follow the same procedure. However, I felt that it would be easier for me to start with sketches, and then to define it, although this was not the case in the book. After several trials I arrived at a draw which looked nice and interesting. I began to wonder how to define it. It wasn't as easy as I thought it would be. I had to break the draw into its components, and think how they relate to each other...I believe this what you meant by creativity…"

[28] PST5: "I also started with a draw, but I used computer software. I begun with quadrangle, and tried to build various other figures on its sides. I dragged the figures and rotated them. While doing so I begun to think that I would never be able to define the figure I draw. Something in the 'dynamism' of the figure bothered me. Shouldn't it be static? Are there many figures on the screen or merely one figure in various situation?...Yes, I believe I was thinking creatively, because the entire process was new to me, and I was the one who asked the question as well as the one who answered them. There was no one to guide me".

[29] PST7: "I don't know if what I did can be considered as creative thinking, because we didn't reached any agreement as to its meaning, but no doubt I was thinking differently. When I started I was curious to see whether the assignment you gave us was possible to accomplish, because it seemed strange at the beginning...So perhaps one has to be motivated by some unusual external stimulus in order to begin thinking differently".
The PSTs' next assignment was to find as many properties as they could for their new defined concept, and formulate relevant theorems. The PSTs were advised to use dynamic computer software in order to ease their work and increase the likelihood of arriving at interesting conjectures. I emphasized that they should not worry about being unable to prove their conjectures, and that in such cases we will make a collaborative effort to prove or refute their conjectures. The PSTs were given five weeks to work on this assignment. The PSTs were also asked to reflect on the entire process, with a deliberate attention given to creativity in mathematics.

THIRD PHASE – FINDING THE PROPERTIES OF THE CONCEPT

Five weeks had past and the PSTs were ready to present their work. Space limitations permit only a general description of the PSTs' work. While presenting the work the PSTs explained what they were trying to do, their failures and successes, their findings and their confusions. Most of the theorems they formulated referred to special lines of the figure, to its circumcircle or inscribed circle and to relations between areas and perimeters. However, for me the PSTs' reflections on the process were no less important than their products. Following are few parts from their reflections:

[30] PST_{12}: "...I didn't take an active part in any of the first two lessons in which we discussed the issue of creativity. I don't want you to interpret my silence as a lack of interest. I just had no idea what to say. This is the first time I am going through such an experience. You gave us an opportunity to see the beauty of mathematics and to develop our ability to do something new, different, and unusual. In that sense it was really a creative doing…"

[31] PST_{7}: "...I think working creatively is concerned with fun. For me the entire process was enjoyable and exciting, and this was the thing that motivated me. Isn't it what creativity about? I think it is. Enjoy while creating and love your creations. These are the feelings I want my pupils to sense…".

[32] PST_{9}: "...I feel like I am at the middle of the process. I still have many things to explore and discover regarding my 'QUADZOID' [the name PST_{9} offered for the concept she invented]. I know I will keep trying to prove my conjecture, although I had already gave you my work. I will do it not for the mark, but for myself. I am really curious to know whether it holds for all QUADZOIDs…and curiosity is very much like creativity. One cannot be creative unless s/he is curious about what s/he does".
EXAMINATION OF THE THREE-PHASE PROCESS

Examining the process the PSTs experienced, I asked myself whether they had really undergone a meaningful process which contributed to the development of their creativity in mathematics. Since the experience was very short, I did not examine its long-term impacts, and the examination of the process is based upon the PSTs' utterances, works and reflections.

**Examination of the first phase.**

The episode described above includes many utterances that go hand in hand with the research literature. Here are a few examples: (i) At the beginning of the process the PSTs distinguished between 'general creativity' and 'creativity in mathematics'. This distinction is in line with Csikszentmihalyi's (1999) perception of creativity as an interaction between a domain and the individual. Then they referred to abilities such as: (ii) To solve non-routine problems (Schoenfeld, 1992); (iii) To generate original proofs (Lamon, 2003); (iv) To raise imaginative ideas (Tuli, 1982), and (v) To invent new theorems (Chamberlin & Moon, 2005).

The utterances of PST11 in lines [21] and [23] deserve special attention, since they imply on PSTs' perception regarding the manner in which mathematics is (or perhaps – should be) taught and learned in school: rote learning for the purpose of acquisition of knowledge. It is well known that PSTs begin their teacher education training with pre-conceptions and beliefs about teaching, learning and interaction with pupils (Tillema, 1995). These pre-conceptions are originated in what Lortie (1975) describes as "apprenticeship of observation" (as can also be seen from the words of PST9 in line [14]), during which the PSTs consolidated their world-view regarding teaching strategies. Thus, teacher education programs must be designed to uproot conservative impressions whilst assimilating others, if developing creativity in mathematics is one of the aims.

**EXAMINATION OF THE SECOND PHASE.**

The utterances of the PSTs after the first week of their work reflect their feelings and the manner in which they worked. The PSTs started the process with hesitations and doubts. At the beginning they worked within a 'vague environment', wondering what might help them to accomplish the assignment. As their stating move all PSTs made some drawing then they thought about a suitable definition. A further research is needed in order to be able to determine whether images are more meaningful for students while working creatively, or perhaps it was a result of the nature of the assignment.
The fact that the PSTs had to clear their road by themselves was very significant for them. According to their assertions the independent work on unconventional assignment contributed to the development of their creativity in mathematics, however, an external stimulus was needed, at least at the beginning, in order to start the process.

It should be noted that in addition to the development of creative thinking, the process had an additional benefit: Reinforcement of the mathematical knowledge, especially regarding aspects that concern the essence of mathematical definition.

EXAMINATION OF THE THIRD PHASE.

After reading the PSTs' work I asked myself whether the assignments encouraged the development of creative thinking. Firstly, I examined the characteristics of their work: it was self-directed and self-assessed type of learning. There was no seeking for the 'correct answer', and the PSTs did not have to follow any specific algorithm. According to Chamberlin and Moon (2005), such learning contributes to the development of creativity in mathematics. Secondly, I examined their engagement: they exhibited curiosity, interest and intrinsic motivation. Starko (2001) affirms that the greater the intrinsic motivation is the greater is the plausibility of creative discoveries. Motivation depends on the environment teachers create in their classrooms. Therefore, thirdly I examined the learning environment. Obviously there was a class environment which did not emphasize the importance of arriving at the right solution, or the application of sequence of rule and algorithms, characteristics of an environment that inhibits the development of creativity (Pehkonen, 1997). Finally, I examined the PSTs' products and reflections. Although some researchers (e.g. Swede, 1993) believe that creativity is a process that results in an outcome that must be universally recognized, I believe that in order to develop creativity in mathematics the outcome must first be appreciated by the learner himself. Developing the ability to examine the value of a product is a part of the broader process of developing creativity. Clearly the PSTs' products are not anything worthy of attention from the community of mathematicians. The important point is that they were given the opportunity to work 'like mathematicians', and as they claim - to enjoy the process of creation, and to be satisfied with their products.

CONCLUDING REMARKS

Creativity is difficult to develop if one is limited to rule-based applications without recognizing the essence of the problem s/he has to solve (Mann, 2006). Many teachers emphasize algorithm, speed and accuracy rather than working towards developing their pupils' creative applications. Failing to encourage creativity in
Promoting Creativity:  
The Case of Creating A Definition

Promoting Creativity:  
The Case of Creating A Definition

mathematics classroom implies missing the opportunity to fully develop mathematical understanding. However, in order for teachers to be able to appreciate the beauty and creativity of mathematics they should better explore the world of mathematics by themselves before they can help their pupils do the same (Pehkonen, 1997). If PSTs would experience teaching methods that are different from the ones they themselves usually experienced as pupils, it is more likely that they would implement them later as school teachers (Beswick, 2005). Therefore, teacher education programs should provide PSTs with opportunities to develop their own creativity in mathematics, reflect on what they do, and together consolidate appropriate ways for developing their pupils' creativity.

In this paper I described one of many possibilities and methods to develop PSTs creativity. Obviously, one experience is not enough. Creativity needs time to develop, since it requires long period of work and reflection (Mann, 2006). Therefore PSTs must be educated regularly within learning environments that promote creativity.

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Abstract

This paper is a review of a large number of research publications. We have analyzed the research to reflect on our years of observation about the gifted and talented students, their creative abilities and skill building in the area of mathematics. We conclude that while giftedness does not necessarily imply creativity, nurturing creativity in gifted students will result in a fulfilling experience for the students and allow them to reach their highest potential. We also deduce that cultural differences play a surmounting role in skill building in mathematical science, which is intertwined with creativity and further linked to giftedness.

WHY MATHEMATICS?

WordNet, a lexical database for the English language developed at the Cognitive Science Laboratory at Princeton University describes Mathematics as a science (or group of related sciences) dealing with the logic of quantity and shape and arrangement. It is recognized also as a science of problem solving. Since problem solving is a creative process (Piggott, 2007), the science of mathematics automatically becomes one of the paradigms for cultivating and nurturing creativity.

Mathematics is as much about identifying and posing problems as it is about problem solving, it is about devising the best and efficient solutions and about applying skills acquired over a range of time. Mathematics therefore provides a medium to foster an environment that encourages creativity and exploration. A powerful tool in supporting students as independent, creative thinkers is the use of questioning (Piggott, 2007) and once again, problem solving in mathematics provides opportunities to students for asking questions and modifying their solutions. Mathematics brings out the attitudes of persistence, risk taking, independence and curiosity. All of these are viewed as creative attitudes (Mildrum, 2000).
GIFTEDNESS AND CREATIVITY

To begin with, we will characterize giftedness and creativity. There are marked differences between the two. Every gifted child does not necessarily have creative abilities while a creative person is not always gifted. Creative abilities emerge in different individuals, in various degrees and manners (Fishkin & Johnson, 1998). Creativity should therefore be viewed as a multi-faceted and complex phenomenon rather than as a single characteristic capable of a precise definition. For example, creativity is viewed as developing ingenious artifacts or solutions, being open-minded and to have an ability to express ideas clearly. In (Mildrum, 2000) the authors link a creative attitude with risk taking, independence and curiosity. In (Fishkin & Johnson, 1998), creative behavior is also viewed as a process resulting in a product unique to the individual who produced it. The cognitive component of creativity is specifically linked to general knowledge, domain-specific knowledge and skills (Seo & Kim, 2005). This is more critical in structured and objective disciplines such as mathematical sciences than in other disciplines. The reason stems from the fact that prerequisite knowledge and skills are imperative ingredients for continued success in mathematics and related fields.

On the other hand, gifted students are identified as having high level thinking skills, linked with highly developed intuitions. In (Clasen, 1979) it is noted that the gifted students can learn more, faster in more depth and breadth, and with more control over what and how they learn than can the more statistically average student. According to (Cho et al. 2000) while gifted students work on a mathematical problem they perceive mathematics analytically, as well as synthetically; in other words, gifted students view mathematical problems as a composite whole - different from their average peers. The gifted students thoroughly investigate the problem, which may suggest that they enjoy working with mathematics.

Clearly, harnessing the creative abilities of gifted students will produce in unbelievable results in academic and creative achievements. Problem solving is a creative process and the same process can in fact be used to develop creative abilities in gifted students. It enables individual ideas to thrive, encourages experimentation, and that gives opportunities to experience and develop in the mathematical space. Even so, one of the primary prerequisite for fostering an interest in mathematics and further cultivating the creative attitudes of the gifted and talented students is developing a strong skill base.

THE CULTURAL ASPECT AND ITS EFFECTS

According to a widely accepted Western definition for the Gifted, presented by the Office of Educational Research and Improvement in the U.S. Department of
The Gifted And Talented: Creativity, Cultural Perspectives and Skills

Education (Neihart, 2001), the gifted children and youth exhibit high performance capability in intellectual, creative and/or artistic areas, possess an unusual leadership capability, or excel in specific academic fields. In this definition, there is little focus on concrete subject areas related to mathematical sciences. While in the eastern cultures, the concept of gifted epitomizes mathematics and science. For example, the Korean gifted education has focused primarily on math and science and those departments are highly interested in creativity because ingenuity in those fields is tied to fiscal prosperity and competition within the global economy (Seo & Kim, 2005). There is a concentrated effort to provide a strong skill base in mathematics at elementary and secondary schools. The same is the case in other eastern countries- China and India that produce the largest number of engineering and science graduates. In fact, China labels mathematics as the mother of all sciences (Jing, 2007).

There are clear patterns of cultural difference with respect to attitudes and perceptions of Mathematical sciences between the eastern cultures and the United States that emerge from the literature review that we conducted. In the U.S. creativity is viewed as a state of personal fulfillment and the understanding or expression of inner sense of ultimate reality, while the Chinese culture views creativity as a person’s ability to generate creative products while emphasizing self-regulation and discipline (Seo & Kim, 2005). In the collectivist cultures of east Asia, education is first and foremost as a means of socialization and this suggest why effort is valued over ability (Cheng, 1998). Cultures, such as in the U.S., where individualism dominates tend to view education as a means of empowering children. Students have choices for courses to pick at the middle school. The choices are influenced greatly by cultural perceptions and trends. Trends, that have not favored mathematics for the last several decades (U.S. Department of Education, 2003; Toppo, 2004).

The education system in the US tends to honor individualism and originality. Such norms are also recognized by the legal system. There is an expectation that the educational system will adapt to individual needs. In fact, the idea of ‘differentiated learning’ stems from these accepted norms. In special education, individual class plans are drawn up for every student (Cheng, 1998) and teachers certified in ‘special education’ are responsible for helping these students succeed. Yet, teachers for the students identified as ‘gifted and talented’ in the US, need have no formal requirements or certifications. Very often they lack pedagogical, as well as content level skills to work with the exceptional students. On an average, the mathematics and science teachers in the US take considerably less college level courses in the content area, and consequently are less prepared as compared to teachers in China and other Asian countries. Additionally, the society’s broad respect for mathematics and science, have a centuries-old lineage in China (Cavanagh, 2007). This results in a system that supports a strong background in
mathematical sciences and higher expectations from the students in China, in general. The same is true for many other Asian countries including India and Japan.

While we are focusing on Gifted students, it is worth noting that the US has scored poorly in mathematical abilities of students at the elementary and secondary level and the result has reflected in fewer American students who are ready to, or choose to pursue mathematical sciences after High School (Toppo, 2004). Even though mathematically gifted children emerge as early as preschool (Johnsen & Kendrick 2005), it is for the education system to nurture the gifted talented and to provide the creative endeavor to thrive. Without the basic skills and the right attitudes towards mathematics, there is little progress in Mathematics in sight the gifted students in the United States.

FURTHER RESEARCH

Continued development of a strong skill set for the gifted students at the middle/high school, along with nurturing their creative abilities is a key to good education and strong foundations in mathematics. Cultural perspectives play an important role. Building on our literature review, we propose to investigate further the factors that most influence mathematical abilities and choices that the gifted and talented students make in the U.S and compare the same with students from selected Asian countries.

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DOES IT MAKE A DIFFERENCE?  
STUDYING THE IMPACT OF TEACHERS' PROGRAM  
AIMED AT MEETING MATHEMATICALLY  
TALENTED STUDENTS' NEEDS

BRURIA SHAYSHON AND BERTHA TESLER  

David Yellin College of Education, Jerusalem

Abstract

This paper presents a questionnaire for studying the change-processes that teachers and student-teachers undergo during a year-long course. The course is designed to assist participants in advancing and fostering mathematically talented students in their daily work in mixed-ability classrooms. The questionnaire addresses three main topics that might shed light on the participants' development: their sense of efficacy; their attitudes, beliefs and thoughts about the learning and teaching of mathematically talented students; and the ways and strategies they actually use to advance talented students in their classrooms.

BACKGROUND

A program for advancing talented students in mathematics within heterogenic school classrooms is about to begin at the David Yellin College of Education in Jerusalem.

The program includes two parts:

A. Enrichment classes for suitable mathematically talented students in elementary and junior high schools.

B. A course for mathematics teachers and student-teachers designed to train them to teach and advance mathematically talented students within a heterogeneous mixed ability classroom. The course will be held throughout the academic year, two hours every week.
Among the aims of the program is raising the mathematics teachers’ awareness for dealing with the needs of talented students in their classrooms. In addition, the program seeks to provide the teachers and student-teachers with theoretical and practical tools for the advancement of talented students.

**Studying the Program’s Impact**

A research study will accompany the program and will examine different aspects of the program. A central goal of the research will be to ascertain to what extend the course affects the teachers’ and student-teachers’ views, efficacy, attitudes and beliefs in relation to the course’s subjects.

**METHODOLOGY**

This research will be carried out among other things by means of a questionnaire – in which many of the questions are in a statements form with Likert Scale responses. This questionnaire was especially designed to provide the formulators of the program with such essential data that will make possible changes and adjustments in real time.

The questionnaire will be answered by the teachers and student-teachers at the beginning and at the end of the course. A shorter version of the questionnaire will be introduced in the middle of the course.

A group of teachers with similar characteristics who will not participate in the course will be asked to fill the same questionnaire. The purpose of this is to study whether and to what extent, the participating teachers group is different from the non-participating teachers group.

**TOPICS OF INQUIRY**

The questionnaire includes questions in several domains that are relevant to teaching mathematics to talented students, such as teachers’ self-perceptions, their views about their talented students, and the ways in which these students’ needs are addressed by these teachers. The three main subjects presented in the questionnaire are:

- Self-efficacy and general teaching efficacy
- Attitudes, views and beliefs about mathematically talented students
- Enrichment and nurturing of mathematically talented students.
SELF-EFFICACY AND GENERAL TEACHING EFFICACY

Albert Bandura (1977, 1986, 1994) defined the concept of self-efficacy as: "People's beliefs about their capability to produce designated levels of performances that exercise influence over events that affect their lives" (1994). Since that time, research in many arenas has demonstrated the power of efficacy perceptions in human learning, performance and motivation.

Gibson & Dembo (1984,) have extended the meaning, suggesting two sub-concepts: general teaching efficacy which relates to how teachers view the connection between teaching and learning and the effects of external aspects on students' learning; and teachers' self-efficacy which relates to teachers' conceptions of their capability for advancing their students. A third term that can be derived is learning efficacy that was found to be connected to self-orientation for learning and pursuing of achievements (Garcia & Pintrich, 1991, 1994).

Three aspects of efficacy are presented in the questionnaire:

General teaching self-efficacy: This examines the teachers' perceptions about their sense of efficacy regarding teaching in general. The questions were first introduced in a Gibson and Dembo (1984) questionnaire with statements like "When a student is having difficulty with an assignment, I'm usually able to adjust it to his/her level".

Talented students teaching self-efficacy: This examines the teachers' confidence in their ability to address mathematically talented students' challenges, in statements like: "I'm afraid that I will not be able to answer talented students' questions".

Learning self-efficacy: This studies the teachers' readiness and willingness to invest time and emotional resources in learning to deal with non-routine and complicated problems with their students, in statements like: "Complicated mathematics problems challenge me".

ATTITUDES, VIEWS AND BELIEFS ABOUT TALENTED STUDENTS

Teachers' beliefs and images are found to be directly connected to their practice of teaching (Calderhead, 1996; Marton & Booth, 1997). This part relates to three areas that are most relevant to talented students. The questions here examine teachers' views about:

Teaching and learning of mathematically gifted students: Statements like: "Mathematically gifted students should be allowed to learn mathematics in separate classes so they can realize their high ability" or "Mathematically talented students almost do not need a teacher to teach them".
A moral stance about mathematically talented students' advancement: This examines the teachers' moral considerations about talented students with statements like: "The limited amount of teacher's energy should be directed to students with difficulties rather than to mathematically talented students" or "The fostering of mathematically talented students should not be a part of teacher's duties in an ordinary class".

Who might be a mathematically talented student: According to Pentago & Birch (1959), in many cases teachers choose the diligent students as talented ones and not the undisciplined ones who frequently might be more talented. The questionnaire relates to some biases of teachers' judgments (Rosmarin, 2001) about who could possibly be a talented student, with statements like: "The possibility that a student with learning disabilities might be mathematically gifted is very low".

ENRICHMENT AND NURTURING MATHEMATICALLY TALENTED STUDENTS

This part of the questionnaire relates to what is actually done as reported by the teachers. Open questions are presented about ways teachers use to foster mathematically talented students in their classroom in an open question like: In the course of your teaching, do you relate in any specific way to mathematically talented students?

Another question examines the capability of teachers for identifying and evaluating creative and elegant solutions and their suggestions as to several possible ways of solving a problem. The questionnaire presents a problem and two possible solutions. The teachers are asked to address and evaluate these two ways of solving the problem, to suggest other ways of solving it, to indicate whether presenting the students with different solutions has any advantages, and if so, what these are.

The proposed questionnaire aims to study the impact of the new program described above. Nevertheless, it is suggested as a general tool to study the impact of any teachers' program aimed at meeting mathematically talented students' needs. In our presentation at the conference we will bring preliminary data gathered from the two groups of teachers as well as our interpretation of the data. In addition we will address the issues of reliability and validity of the questionnaire.

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ENHANCING TRANSFER AS A WAY TO DEVELOP CREATIVITY WITHIN THE DYNAMIC STRUCTURAL LEARNING

FLORENCE MIHAELA SINGER

Institute for Educational Sciences, Bucharest, Romania

Abstract

In the context of the dynamic changes of today’s society, teaching for enhancing transfer is an educational challenge. The dynamic structural learning aims at stimulating transfer capacities in students through developing dynamic mental structures. This type of learning supposes a specific curriculum that consists in training mental operations applied to information on various levels of abstracting, simultaneously with transfers in between. A longitudinal experimental study that promotes dynamic structural learning shows that an adequate methodology can develop creative approaches in problem solving and posing.

THEORETICAL FRAMEWORK

The literature on cognition frequently approaches the theme of creativity. From “big C creativity” – the creativity of personalities that have changed the history of a domain, to “small c creativity” – of an everyday context (Gardner, 1994), the analyses focus on expert’s thinking, novice’s thinking, and transfer abilities within the problem solving process of various categories of individuals (Csikszentmihalyi & Wolfe, 2000). A large body of research is devoted to studying creativity in school context. Hadamard (1945) assumed that creativity in mathematics requires an intuitive mind and time for reflection and incubation of ideas. From the teaching perspective, the challenge is to provide an environment of practice and problem solving that stimulates a diversity of approaches, while avoiding the imposition of problem-solving heuristic strategies (Pehkonen, 1997). The open-ended problems allow students to experience stages of mathematical creativity and to develop higher-order mathematical thinking (Becker & Shimada, 1997). Transfer reorganizes thinking by broadening the application of acquired concepts and behaviors. Considerable research shows that transfer is hard to come by,
particularly far transfer (e.g. Perkins & Grotzer, 1997; Dunbar, 2001). However, a large body of experimental studies argues that transfer can be, and has been, attained when the conditions of learning foster reflective abstraction, and practice on deliberately diverse cases. For example, the heuristic instruction program (Schoenfeld & Herrmann, 1982) to teach problem solving to college students covered control and managerial strategies, bringing into class teaching elements of modelling, coaching, and scaffolding, as well as problem solving in small groups and with the whole class. Studies show that, in such cases, abstracted representations do not remain as isolated instances of events but become components of larger, related events, or schemata (Gick & Holyoak, 1983). Students are more likely to transfer thinking skills if they are motivated to use the skills they have learned. The role of the teacher in promoting creative learning and transfer involves creative environments and includes perspectives on representation and affect (Goldin, 2002). Bringing representational models to school early in an informal way stimulates abstraction during cognitive development (Goldin, 1998; Singer, 2003). Teaching for representational change (Singer, 2007a) facilitates transfer by avoiding rigid connections. Children employ various layers of understanding and bridging techniques, especially the use of multiple representations to increase problem solving abilities, and they can be stimulated to do so by educational tools and practices.

**DYNAMIC STRUCTURAL LEARNING: BASIC CONCEPTS AND PROCEDURES**

The awareness of the dynamic variation in cognitive development (Fischer and Bidell, 1998; Yan and Fischer, 2002) can help in designing learning models that create an environment in which students apply relevant knowledge and skills to the solving of real problems (Kilpatrick, 1987; Renzulli, Gentry & Reis, 2004). Achieving such abilities supposes a specific mental training. In this area of research is situated the dynamic structural learning experiment. In a synthetic description, the dynamic structural learning (DSL) of a domain is seen as a bipolar cognitive construction: on the one hand, the domain to be taught is organically integrated in a constructed structure, which is focused on developing a specific way of thinking; on the other hand, learning each subgroup of the structure implies an active reconstruction of its meaning (in the sense of the constructivist definition of learning). The dynamic structural learning process (Singer, 1995) supposes structuring the content that is learned (i.e. constructing a specific curriculum) and structuring the training in order to develop in children abilities analogous to the ones of the expert. Emphasis is placed on creating learning situations where students are doing, thinking, and feeling explicitly what mathematicians do (often implicitly) as a means of developing mathematical creativity (as opposed to replication and repetitive practice). From a pragmatic view, the DSL model
Developing Creativity Within
The Dynamic Structural Learning

supposes to train mental operations applied to information on different levels of abstracting, simultaneously with transfers and crossings in between. These generate dynamic mental structures at the cognitive level. Teaching within a DSL program allows an operational focus that moves beyond tasks, domains, and cross-domain connections in order to develop dynamic knowledge structures (Singer, 2001). The building of dynamic mental structures implies practicing categories of transfer operations. These are shifts from one level of abstracting to another, shifts from one operation to another basic or composite operation, as well as crossings among various chunks of information. A few examples of the categories of tasks used in DSL to foster creativity are given below. These involve the technique of generating.

- **Generating through counting** (“Count by threes starting with 2. Vary the counting step.”)
- **Generating through estimation** (“Give examples of exercises the result of which is closer to 500 than to 600.” “Give examples of objects the height of which is about 2m.”)
- **Generating through classification** refers, on the one hand, to finding as many classes as possible for a given object, and, on the other hand, to finding as many representatives as possible in a given class (for instance, to enumerate as many different objects as possible that can have the same use); (“Draw geometrical figures in which a triangle is formed by: the diagonals of a polygon; two sides and one diagonal of a polygon; the intersection of two polygons; the face of a pyramid; the base of a prism, etc.”; “Find ten natural numbers that are multiples of 2 and 3 and not of 12.”).
- **Generating through substitution** (“In the formula \(a+7 = x\), \(x\)-unknown, replace \(a\) by different values and devise problems appropriate to the formula obtained; replace \(a\) through an operation of subtraction and devise problems fitting the formula.”)
- **Generating through selection-discrimination** (“Draw different polygons in which the diagonals are perpendicular; draw different polygons in which diagonals are not perpendicular.” or “Giving a ring with the diameter of 5, describe some 3D shapes that may cross this ring.”)
- **Generating through increasing and decreasing** (“Consider the problem: “There were 15 trees. Other 7 were planted. How many trees are there?” Add another piece of information so that the problem may have a solution according to the formula: \(a + b - c = x\), \(x\)-unknown.”)

Similar examples were developed for: generating through representation, generating through interpretation, through analogy, substitution, comparison,
selection-discrimination, combining, arranging, generating through changing (varying) the hypothesis, or through changing (varying) the conclusion, etc. (Singer, 2001).

**RESEARCH QUESTIONS AND METHOD**

The research presented in this report was driven by some specific questions addressing creativity: Do the transfer tasks as the ones presented above enhance creativity? Can creative capabilities be stimulated in primary grades through a systematic training? Does dynamic structural learning prove effective in enhancing creativity when learning mathematics?

To probe the effectiveness of the DSL model, a 4-year longitudinal study of teaching and learning mathematics was conducted in primary school. The number of children in the classes varied between 26 and 34, and in total, 232 children in nine experimental classes were involved. The experimental program tracked cohorts of children from Grade 1 to Grade 4 (aged 6 – 7 to 10 – 11 years). Preparatory meetings and feedback sessions were conducted with the teachers involved in the experiment. The teachers received detailed description of the tasks that they were going to offer to students, and the teaching periods were followed by discussions, on a weekly base. The description of the learning activities is contained in four teacher’s guides (Singer & Radu, 1994-1997). In short, the technique consists in activating creative abilities by resorting to tasks organized as games. A few examples are given below, for grade 2.

- **Starting from isolated numbers.** A child proposes a small number, for example 3, and asks a colleague to develop a number sequence starting from 3. Then the teacher or a child asks that the same number 3 be the counting step of a sequence between 12 and 30, or between 60 and 90, etc. Another game requires the construction of a number in which the number 3 is a compulsory component, for example 436, or it is not a component, for example 456. Children are stimulated to propose many such examples. Next, problems are composed starting with the number 3. (e.g.: Start from 3 and add its double: what is the number you get? etc.; 3 chicken eat 3 worms each … etc.; Use 3 coins to compose a given amount of money, etc.). The same procedure is carried out starting from other numbers and developing serial arrangements, or practicing comparisons, estimations, compositions and decompositions of numbers, and problem posing.

- **Starting from exercises.** A simple exercise is chosen and the task is to device similar exercises. The starting exercise is compared with others (concerning the number of terms, operations, etc.) in order to highlight similarities and differences. Then the initial exercise is transformed into a problem. New
Developing Creativity Within
The Dynamic Structural Learning

proposals advance exercises in which the number of terms and number of operations are increased. For each exercise, a variety of word problems is generated keeping the same data, but changing the context.

- **Starting from word problems.** A simple problem of addition (of the type \( a+b=x \), \( x \) – unknown) is proposed. The problem is reformulated keeping the same numbers (for example, by changing the position of the question). The problem is extended as to contain two or three operations of addition, or subtraction or a combination of addition and subtraction. Constantly, the initial problem is compared with others in order to uncover similarities and differences.

Children are to practice the tasks succinctly described above in a gradual progression of internalising: orally, silently (in mind), in writing (without or with minimum verbalization and the result is required for checking). Letters are used just accidentally, or gradually, depending on the students’ level and the teacher understanding regarding their appropriate use.

**RESULTS**

The practice of various transfer tasks in consistent mapping contexts, in which distractors gradually interfere in a field of invariant conditions, leads to the development of specific habits of mind (Palmeri, 2002; Singer, 2006). Within the DSL, the classroom practice covers cycles of systematic, randomized and structured training that leads to the development of dynamic mental structures. In this type of experiment, measuring the results becomes more complicated for various reasons, such as: the difficulty to measure the learning of a certain capacity, the presence of many disturbing factors in the classroom, the difficulty to create control classes that satisfy requirements similar to the experimental ones, the relativity of the information obtained through written tests in the circumstances of non-homogeneity and non-sampling (as the teachers participated in the experiment on a volunteer basis). Taking notice of the above reserves, the sources for collecting data were: classroom observations, interviews with the teachers involved in the experiments, and tests. The classroom observation was considered essential for drawing conclusions because it allows, in addition, to ascertain individual and group reactions and to evaluate motivation, interest, spontaneity, and the general atmosphere in the class. During the school year, the children in the experimental classes were tested 15 times. Each test contained 10 items, among which four to six were open-ended questions focused on creative tasks. A few examples of the types of these creative problems and the percentage of correct answers are given in the table below. The data refer to grade 3.
Table 1: Percent of correct answers in experimental and control grade 3 classes for a few items

<table>
<thead>
<tr>
<th>Type of creative problem</th>
<th>Percent of the students in the experimental classes who gave correct solutions</th>
<th>Percent of the students in the control classes who gave correct solutions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Finding consecutive terms when knowing their sum; building (implicitly) a model for this problem</td>
<td>75%</td>
<td>26%</td>
</tr>
<tr>
<td>Generating numbers whose digits fulfill certain condition; finding all the variants</td>
<td>62.4%</td>
<td>45%</td>
</tr>
<tr>
<td>Finding the missing digits in a multiplication; using the inverse operation</td>
<td>67%</td>
<td>33.5%</td>
</tr>
<tr>
<td>Creating word problems when some conditions are given</td>
<td>80%</td>
<td>35%</td>
</tr>
</tbody>
</table>

The average success at solving creative tasks was more than 60% of the students for each of the classes involved in the experiment. In order to put these results in the context of usual teaching, I refer to an evaluation made on a representative sample in grade 4 at the end of the same school year. This assessment involved 18,844 students from 992 schools in urban and rural areas. One of the items requiring a non-standard creative answer in this test was: “Compose a problem using numbers smaller than 20, which is solved using addition, multiplication and subtraction”. This item was correctly solved by only 20% of the students (only the text of the word problem was scored, not its solving), and 46.1% of the students did not attempt the task at all.

Beyond the quantitative data, some qualitative aspects revealed by the classroom observations seemed to be more significant for the experiment. Thus, while concerning computing, the outcomes were quite similar in experimental and control classes, significant differences have been identified in problem solving activities. Most students in the control classes encountered difficulties such as: the differences between the data and the topic were not correctly discerned, the useless data were frequently taken into account, the missing ones were not noticed. Contrary to these, most students in the experimental classes showed awareness of the text meaning, facility and desire to device problems, and a preference to explain through concrete objects or other models, making less significant language errors.
CONCLUSIONS

The national assessment discussed above raised concerns about the big percentage of the students than refused even to try a solution (46%) for a creative task. However, when this mass refusal is compared with children’s preference for multiple operations in the experimental classes, we can conclude that a specific training might stimulate and mobilize children’s native creativity in school problem solving. Another conclusion of this experiment is that teaching that enhances transfer and creativity has the potential to enhance also the capacity of logical and efficient learning. Children become able to find alternative representations, to predict outcomes, to note failure in understanding, to come back or to plan ahead in order to improve their own knowledge. Still, research is needed in this area to find an adequate balance between stability and flexibility of thinking within the cognitive development.

REFERENCES


A PERSONALIZED ANALYSIS OF STUDENTS’ ANSWERS IN MASS COMPETITIONS – SOURCE FOR COMPREHENSIVE ASSESSMENT AND THE PROMOTION OF CREATIVITY

FLORENCE-MIHAELA SINGER
Institute for Educational Sciences, Romania

CRISTIAN VOICA
University of Bucharest, Romania

The most popular mathematics contest in the world, the Kangaroo competition of applied mathematics, gathered more than 4,000,000 participants from 33 countries in 2006 and a roughly similar number in 2007. Conceived as a competition for all, it is addressed to students aged 7 to 18, at all levels of education, all levels of abilities and all types of schools. Given its large base of participants, this contest can be used as a way to identify giftedness within and beyond classroom practice. In Romania, 247,000 students from 3,800 schools participated in the 2007 competition.

The Kangaroo tests, consisting of multiple-choice questions of increasing difficulty, are meant to promote students’ creativity. The tests are proposed by teams of mathematicians from all participating countries, who pay attention to an intersection of national curricula, focusing in the same time on devising problems that are different compared to the usual ones in the textbooks and auxiliaries used in each country. Moreover, there is a vertical tracking of the problems, in order not to repeat a type of problem already used in a previous year. The contest efficiently values the multiple choice system because the proposed problems are idea-based and not computational; they need a short response-time, but they suppose a special intuition to discriminate among the variants of answers; moreover, the students avoid filling in at random because it is a penalty for each wrong answer. In addition, as the problems count differently in the final score of each test (being classified on three levels of difficulty), the students have to develop a global strategy of approaching the test – therefore, the ones able to exhibit metacognitive skills have more chances to get superior results.

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Haifa, Israel, February 24-28, 2008
So far, the Kangaroo competition has led to the accumulation of huge databases of students’ answers. To obtain pertinent information from these databases (that enrich year by year), we have developed a classification system of the problems and a system of reporting each student’s paper to various statistical clusters in order to communicate information about personal level of mathematical competence, as well as preferences and predispositions.

PURPOSES OF THE PROJECT

The following objectives have driven the design of the project:

1. Develop a digital operational system of processing the databases obtained from mass competitions, which can communicate significant information to various actors in the educational system (students, teachers, researchers, decision makers)

2. Offer students a detailed criterial analysis of their answers with the purpose to help them to develop their own learning techniques and metacognition. The analysis supposes to report individual performances to the performances of the other participants of his/her cohort based on clear established criteria.

3. Offer teachers a tool for knowing students’ potential, obtained through the automatized analysis of a huge volume of data. Through the Internet, teachers can access structured information or they can ask for specific analyses regarding the students from their class/school.

4. Find out what the students know, mistake, prefer. Find elements that differentiate ways of mathematical thinking of various groups/categories of students. The distractors of the proposed problems have specific meanings, which can give information about the types of solving strategies adopted by students.

5. Collect and correlate data from tracking students’ results along multiple competitions.

PROJECT TOOLS

We have developed and used the following tools in order to reach the project aims:

1. Specialized hardware and software for recording and processing data

2. Conceptual and operational tools that ensure the classification of problems (Singer & Voica, 2002, 2006) and the construction of meaningful distractors for interpreting students’ errors
3. Databases of problems conceived so that they stimulate students’ creativity, problems that are developed by the cooperation of teams from many countries

4. Databases of students’ answers, which allow a systematic analysis of both students’ competences and of the proposed problems

5. Database of involved students’ and teachers’ opinions obtained through interviews

6. Case studies obtained through video recording some students during the tests.

PROJECT MATHEMATICAL CHALLENGES

The project supposes a careful approach concerning the mathematical content (problems, the process of classifying problems, alternative solutions, solving indications with various degrees of detailing, distractors, etc.) as well as concerning mathematics learning (problem solving difficulties, mistakes and their generation sources, convergent thinking training, divergent thinking training, etc.). Consequently, the project has the following mathematical and educational challenges:

1. Analyzing sets of problems from various perspectives regarding intrinsic criteria (concerning problem classification), as well as extrinsic criteria (concerning the characteristics of various samples). These allow organizing the databases of problems in order to be used for efficient learning.

2. Designing an individualized mathematics learning framework based on knowing each student’s capacities (strengths, weaknesses, preferences). The teacher can use this information to harmonize the teaching-learning process to students’ needs and increase motivation for mathematics learning.

3. Recording students’ progress on various criteria in order to make a prognosis about his/her future development, career, continuous education.

4. Identifying common points/ differences in educating various categories of students; selecting students/ teachers with extreme results and analyzing in detail these situations in order to identify the sources/ causes/ elements that make the difference.

5. Extending students’ assessment on the internet and offering a consistent feedback, without direct human intervention. Based on the classifications made in the databases, the student receives lists of structured problems following certain criteria that allow improve problem solving abilities. The software allows a driven learning through getting gradual hints for solving.
OUTCOMES OF THE PROJECT

The following outcomes are envisaged:

- Personalized analysis regarding problem solving capacity of each participant student. This might lead to increasing motivation for learning.
- Survey regarding the level of acquisitions in mathematics for various clusters (students, schools, communities). These surveys can be useful to research and to the decision making process.
- Complex databases that can provide categories of problems for training certain capacities of students. These databases can be used in comprehensive assessment and in improving learning.
- Human resources that use these databases (teachers trained for organizing competitions and proposing problems structured on certain criteria, researchers using databases for scientific research, school psychologists offering adequate counseling based on the gathered information).

USING THE DATABASE: AN EXAMPLE

The outcomes of the project allow to develop comparative analyses and to draw conclusions for research in mathematics education. We present below a short excerpt from a vertical processing of students’ answers to the following question (a more detailed analysis is in Singer & Voica, submitted):

*Observe the figure below. Every time, at the next step, the black shapes are divided into three equal parts. What is the next number?*

A) 17;  B) 23;  C) 31;  D) 45;  E) We cannot continue, because the black parts become too small.

Some data about the answers to this question by a sample of 143,808 students are given in Table 1.
Comprehensive Assessment and the Promotion Of Creativity

Table 1: Statistical data regarding the answers given by 3rd to 6th graders to a multiple-choice question

<table>
<thead>
<tr>
<th>Grade/age of the student</th>
<th>Total number of participants</th>
<th>Percent of students choosing answer E</th>
<th>Percent of students answering correctly</th>
</tr>
</thead>
<tbody>
<tr>
<td>III / 8-9 years old</td>
<td>45 934</td>
<td>11.41%</td>
<td>31.13%</td>
</tr>
<tr>
<td>IV / 9-10 years old</td>
<td>42 094</td>
<td>11.96%</td>
<td>42.05%</td>
</tr>
<tr>
<td>V / 10-11 years old</td>
<td>30 583</td>
<td>27.42%</td>
<td>37.31%</td>
</tr>
<tr>
<td>VI / 11-12 years old</td>
<td>25 197</td>
<td>23.61%</td>
<td>43.89%</td>
</tr>
</tbody>
</table>

We note that the percentage of correct answers is roughly similar, slightly increasing with grade. We focus the distracting factor E. Surprisingly, the percentage of choosing the answer E supports a significant increase (2.5 times bigger) for grades 5 and 6 comparing to grades 3 and 4. This is unexpected enough given that with grades 5 and 6, students begin a more systematic study of geometry. Psychological studies show that there is a variation in students’ cognitive achievements at this age, a fact that is confirmed by our sample. However, this is not a sufficiently refined explanation for the significant statistical difference between the 3rd–4th and 5th–6th grades. We claim that a cognitive conflict is activated between the processional and the topological perception engaged through the task, induced by the constraints generated by the instable mental configurations at this age.

REFERENCES


THE CONTRIBUTION OF MOFET PROGRAM TO ITS' STUDENTS' SELF-ESTEEM, SELF-CONTROL AND PERSONAL GROWTH INITIATIVE

OFRA STEIN,
Tel-Hai College, Golan Research Institute, Israel

MIRIT KASTELMAN
Ohalo College, Israel

The purpose of this evaluation was to examine the achievement of the MOFET program with respect to three psychological concepts: self-esteem, self-control, and personal growth initiative. The study included 210 students in the 9th and 10th grades in high schools in the north of Israel. Students completed a quantitative questionnaire addressing the three variables under consideration. The results indicated significant differences between the two groups; for all three concepts, the MOFET students were found to have higher scores than non-MOFET students. It is therefore concluded that in addition to advancing their level of academic achievement, the MOFET program may contribute to the personal development of its students.

BACKGROUND

Assumptions, framework and modes of operation

MOFET (a Hebrew acronym for mathematics, physics, and culture) was established in 1992 by immigrant scientists who sought to advance the level of mathematics and science instruction in Israel. The MOFET association operates in the informal educational framework with approximately 2,500 students.

1 According to MOFET official website: http://www.mofet.org.il
Ofra Stein
Mirit Kastelman

participating in after-school/evening enrichment studies; and in the formal education framework with approximately 11,000 students enrolled in unique MOFET classes offered in 100 schools across Israel.

MOFET strives to foster excellence in both academic and social areas through the use of unique teaching methods. The program’s objectives include: to increase the level of academic and cognitive achievement; increase the number of students who complete high level matriculation examinations in mathematics and the sciences; improve the quality of teaching and develop learning materials; and foster motivation, leadership, and students’ belief in their own abilities. A considerable portion of the program is devoted to developing mathematical thinking, logic, and creativity.

In keeping with the writings of Vygotsky and Froidenthal, learning is based on teaching methods that take a heuristic approach to teaching, the principles of which include graduated progress based on existing knowledge, graphic visualization that advances understanding, experience in research and discovery, as well as coping with high levels of difficulty, accelerated learning pace, systematic work, which foster systematic and structured work habits (Schneiderman et al., 2003). A connection has been found between teaching according to these methods and improved mathematical thinking abilities in students (Koichu, 2001). In the framework of the program, teachers are trained to work according to this strategy; to encourage students’ sense of capability; and to cope with various difficulties, such as competition (Koichu, 2001).

Motivation to learn is considered paramount to the student’s growth, and MOFET students in the secondary school program are required to participate in a preparatory course that tests their motivation, self-discipline, behavior in situations of uncertainty, ability to work in a team, and level of curiosity (Schneiderman et al., 2003). Additional projects are carried out by MOFET as part of its overall efforts to encourage excellence, such as the nationwide Mathematics and Physics Olympics. The Olympics aim to encourage curiosity, creativity, and motivation, as well as to exemplify the contribution of the language of mathematics/physics in problem solving and tear down the psychological barrier that often surrounds the study of science. Another program, the Accelerated Degree Program, exposes students to the academic world and provides an outlet for the talents, curiosity, and intellectual capabilities of high-achieving students, without interfering with their formal study framework. Leikin (2006) found MOFET students to have higher opinions of their capabilities and higher grades relative to their efforts in learning; Horovitz (2007) views MOFET students as an agent for the socialization of excellence.

The MOFET association endeavors to recruit teachers who have suitable academic qualifications, talents, and pedagogical experience. At the same time, MOFET uses
external evaluators to measure the level of instruction, study methods, and teaching outcomes; this provides teachers, regional coordinators, and MOFET directors with professional feedback regarding the program and its educational returns. In the formal education framework, the study subjects and teaching materials comply with the curricula set forth by the Ministry of Education. In enrichment programs, students study according to a program developed by MOFET teachers. Instruction is adapted to the level of knowledge and rate of study of the students in the class.

STUDY VARIABLES

Self-esteem reflects the value that individuals place on themselves, based on personal experiences, including the combination of success and failure in their studies and social situations (Bar El, 1996; Ziv & Ziv, 2001). A direct link has been found between low self-esteem and aggressive behavior toward other children (Moore & Kirkham, 2001). In school-age children, success in studies has the greatest influence on the construction of the child’s self-esteem: students with high levels of achievement and study skills report high levels of learning motivation and high levels of self-esteem, whereas students who lack skills and have low grades report low degrees of motivation to learn and low levels of self esteem (Naflai, 2003; Muris, 2001).

Personal Growth Initiative is defined as the active involvement and guidance of the individual in the personal growth process. This term encompasses cognitive components of self such as beliefs, positions and values supportive of personal growth, and willingness to change (Robitschek, 1998). Personal Growth Initiative parallels to some degree self-motivation. Ryan & Deci (2000) cited three factors that strengthen internal motivation, thus at the same time advancing personal and social development: sense of independence; sense of self-efficacy; and sense of belonging. Deci et al (1991) found that when students are interested in their studies, appreciate the value of learning and have confidence in their own abilities, their internal motivation increases, there is an increase in understanding, and higher levels of achievement, as well as increased personal growth and improved coping abilities.

Self-control indicates the readiness to put off immediate gratification in factor of more desirable behavior (Ronen, 1997). Self-control skills integrate thinking skills alongside social skills that restrain and prevent anti-social behavior: anger management, control of violent behavior, awareness of feelings, and consciousness of behavior (Larson & Turner, 2002). Research literature indicates that the learner experiences overall success and achievement in learning, the learner’s capacity to control his behavior increases (Ronen, 1997; Fishbach, 1998; Rosenbaum, 1993).
METHODOLOGY

This study was carried in April and May, 2006, in two comprehensive (six-year) high schools in northern Israel. A total of 210 students participated in the study. The students were 9th graders (51.4%) and 10th graders (48.6%) from eight different classes. Half of the classes (two per grade level) were MOFET classes (95 students); the other classes (two per grade level) were regular classes (115 students), studying according to the standard curricula and format.

Evaluation Tool

A questionnaire was prepared and used to measure the three variables: self-esteem, self-control, and Personal Growth Initiative. The particulars of the questionnaire were taken from valid reliable tools: Self-Esteem Scale (Rosenberg, 1965) – reliability \( \alpha = .75 \); Social Skills Rating System (Gresham & Elliot, 1990) – reliability \( \alpha = .80 \); and Personal Growth Initiative Scale (Robitschek, 1998) – reliability \( \alpha = .84 \). The questionnaire comprised 20 items on a six-step Likert scale.

Research Hypothesis

The MOFET framework is expected to be more effective than the regular educational framework with regard to each of the variables measured.

FINDINGS

Multivariate analysis of variance (MANOVA) showed significant differences between the MOFET and non-MOFET students' concerning each of the variables, for each grade level (F(3,204)=2796.; p<.001). For all indices, the scores of MOFET Class students were significantly higher than the scores of students studying in regular classrooms, in both the 9th and 10th grades, as reflected in the following charts.
Students’ Self-Esteem, Self-Control and Personal Initiative in MOFET Program

Table: Averages and Standard Deviations for “Personal Growth Initiative”, “Self-Control” and “Self-Esteem” Indicators among 9th and 10th Graders MOFET Classes Students vs Regular Classes

<table>
<thead>
<tr>
<th>Group</th>
<th>Grade</th>
<th>No. Students</th>
<th>Personal Growth Initiative</th>
<th>Self-Control</th>
<th>Self-Esteem</th>
</tr>
</thead>
<tbody>
<tr>
<td>MOFET</td>
<td>9</td>
<td>53</td>
<td>4.21</td>
<td>.60</td>
<td>4.54</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>42</td>
<td>4.13</td>
<td>.67</td>
<td>4.57</td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>95</td>
<td>4.17</td>
<td>.63</td>
<td>4.55</td>
</tr>
<tr>
<td>Regular</td>
<td>9</td>
<td>55</td>
<td>3.80</td>
<td>.93</td>
<td>3.81</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>60</td>
<td>3.82</td>
<td>.96</td>
<td>3.88</td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>115</td>
<td>3.81</td>
<td>.94</td>
<td>3.85</td>
</tr>
<tr>
<td>All</td>
<td>9</td>
<td>108</td>
<td>4.00</td>
<td>.80</td>
<td>4.17</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>102</td>
<td>3.95</td>
<td>.86</td>
<td>4.17</td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>210</td>
<td>3.98</td>
<td>.83</td>
<td>4.17</td>
</tr>
</tbody>
</table>

CMEG-5 241
SUMMARY AND CONCLUSIONS

The general research hypothesis was that the MOFET program, which aims at fostering academic achievement and development of thinking skills, also has a positive influence on the personal-behavioral level: the emphasis on excellence might have impact on students’ self-esteem and develop propensity for development and growth, whereas the focus on exact sciences requires discipline and self-control. The results of the evaluation support these hypotheses: for all three variables, the scores of MOFET students were higher than the scores of their counterparts in the regular classroom.

The result of this evaluation are consistent with previous studies: the encouragement of independence and curiosity can contribute to developing motivation and self-efficacy (Deci, 1991); in the MOFET framework, students also acquire study skills, which have been proven to contribute to the development of a sense of responsibility and self-discipline (Niemczyk & Savenye, 2001).

The program also focuses on the emotional-behavioral plane through its emphasis on developing student’s motivation and belief in their own abilities. Past studies have linked motivation to self-esteem (Naflai, 2003; Harter, 1998; Covington, 2002; Klassen, 2000). In addition, aspects of self-esteem are apparent in the sense of self-efficacy (Bandura, 1977), thus, the investment in the development of motivation and self-efficacy may be partly accountable for the findings with regard to Personal Growth Initiative. Concerning the teaching aspect of the program, the teaching methods relate to Strayhorn’s (2002) recommendations for developing self-control by setting goals and building gradual progress plans, whereas the meticulous selection of the teaching staff corresponds with the ideas of Rowe (2003) about the teacher as the primary factor that forms the students' learning products - on the cognitive, social and personal level.

Recommendations

This study showed the effectiveness of the MOFET program on the psychological plane, and recommends further in-depth study of the program principles so that they can be applied in regular classrooms as well.

Limitations of the Study

The researchers acknowledge that the use of a questionnaire poses a risk to the credibility of the responses given. The research was conducted in only two schools, therefore the validity of generalizing its findings is low. It is recommended that the study be expanded to include additional schools.
The criteria for accepting students to the MOFET framework does not include a particularly high level of achievement or intelligence – only the willingness and ability to persevere and work hard.

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[http://www.mofet.org.il](http://www.mofet.org.il) (MOFET official website)
RESEARCH REPORTS

RESEARCH ON PROMOTING CREATIVITY OF GIFTED STUDENTS – FUTURE TEACHERS IN MATHEMATICS

EMILIYA VELIKOVA

Department of Algebra and Geometry, University of Rousse, Rousse, Bulgaria

E-mails: emily@ami.ru.acad.bg  emivelikova@yahoo.com

Abstract. The paper presents the results of a study on promoting creativity of gifted university students doing successful self research and creating new mathematical knowledge in Geometry with the help of contemporary information technologies.

Key words: gifted students, mathematical creativity, causal-experimental abilities, IT usage.

BACKGROUND

Social, economical and cultural development of every country depends nowadays on the quality of information usage. The changing of characteristics of education (O.Osipova, 2005, p.461-463; V.Voinohovska, M.Theodosieva, 2006, p.388) leads to the necessity to promote creativity of gifted students - creating new scientific knowledge on the basis of contemporary IT usage. One of the main factors for realization of that kind of training in mathematics is including gifted university students, future teachers in mathematics, in self research in the area of mathematics.

The research students’ work is a process for solving open problems. It connects the scientific methods, from one side, and pedagogical aims, students’ characteristics and needs, from another side. It usually includes the next stages: motivating for research work, formulating the open problem, learning theory, formulating a hypothesis, chosing of the research methodology, collecting, systemizing and analyzing information, evaluating the results, making conclusions, presenting the results (A.Leontievich, 2003, p.10).

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The theoretical analysis and many research results with school students (E.Velikova, 2001, p.23-43) shows that the necessary condition for involving students in creative work is that they to be “creative-productive gifted” students who possess or can develop a cross-section of three clusters of characteristics (J.Renzulli, 1978, p.59-63):

♦ Above-average general abilities and specific abilities.
♦ A high level of creativity as: fluency, flexibility, and originality of thoughts, “mentally playful”, openness to experience, curious, willing to take risk in thought and action, etc.
♦ High level of task commitment as: high level of interest, fully involvement in a particular problem or area of study, self-confidence, setting high standards for one’s work, etc.

J.Renzulli postulates “our orientation must be redirected towards developing “gifted behaviours” in certain students (not all students) at certain times (not all the time) and under certain circumstances” (J.Renzulli, 1978, p.63, J.Renzulli & S.Reis, 1997, p.5-15).

The above-average general abilities can be measured by IQ test. The specific abilities for research work depend on the training activities and the scientific areas. They are here:

2. The causal-experimental specialized structural system of abilities which is applied on causal reality structures and includes (C.Constantinos and others, 2003, p.2; A.Demetriou. and others, 1993, p.482):
   ♦ Combinatorial abilities form the cornerstone of the system.
   ♦ Hypothesis formation abilities enable the individual to induce prediction about possible causal connections on the basis of data patterns.
   ♦ Experimentation abilities enable the individual to “materialize” hypotheses in the form of experiments.
   ♦ Model construction abilities enable the individual to property map the results of experimentation with the original hypothesis in order to reach an acceptable interpretation of the data.

The research on promoting creativity of gifted students also observes:
Promoting Creativity of Future Teachers in Mathematics

♦ Special skills: critical thinking and skills for solving problems in the area of geometric transformations.

♦ Basic skills for working with information: information skills, skills for information technologies usage, computer tools skills, communication skills.

♦ Knowledge application.

The “research potential” is called a system of abilities, skills, knowledge and task commitment which are necessary for successful self work of students with aim to create new scientific knowledge in the area of geometric transformations on the basis of contemporary information technologies.

METHOD

The goal of the research is to investigate the possibilities for promoting creativity of gifted university students.

Hypothesis of the research. Applying a psychological-pedagogical model (see below) of training of gifted students in Geometry on the basis of new information technologies sets the pattern for the developments of the research students’ potential and the high qualities of their scientific results.

The “creatively gifted” university students (future teachers in Mathematics) are the subject of the research.

The object of the research are: the development of the research potential of the “creatively gifted” students and the high quality of the students’ scientific results.

THE CONCEPTUAL MODEL FOR TRAINING OF STUDENTS FROM THE EXPERIMENTAL GROUP (EG) INCLUDES THREE STAGES.

First Stage. The general training of students for self research includes:

1. Developing students’ interest and knowledge for transformations by presenting very attractive applications of transformations for solving mathematical problems and offering problem-solving strategies for easy creation of new scientific problems.

2. Developing abilities for combing factors, analyzing of results, etc.

3. Developing of skills for IT usage by searching for interesting historical information and problems that can be transformed into new one’s.
Second Stage. Preparing of students for self research in the area of geometric transformations.

The training team plans all activities with respect to the experimental tasks. The main stages are:

1. Presenting the Median-Dual Transformation and some of its applications.

2. Group working for rediscovering a geometric transformation with the help of a didactical instrument (Application). The instrument is a system of problems which is bases for:
   ♦ Combinations of elements, for example, the elements from Problem 1, Application.
   ♦ Formatting hypothesis about possible causal connections by “causal-experimental thinking” (Demetriou, A. & A.Raftopoulos, 1999, p.8-9) and strategies usage as: analyze, decreasing of elements in the combination and experimenting with them.
   ♦ Experimenting to prove the hypothesis correction by analogy, generalization, analyzing the degree of similarity, improving the system of given data, empirical strategy usage, verification of the results.
   ♦ Modeling of the results – constructing theoretical background of a transformation (definitions, theorems, corollaries), applying it as a powerful method for creating new geometric problems. For example, Problem 2, Problem 5, Problem 7 and Problem 8 (Application) lead students to discover some important qualities of the constructing transformation, stable points, application of composition of transformations.

The didactical instrument is also used for developing students’

- creativity (Problem 3, Application),
- basic skills (Problem 4, Problem 5, Application),
- tasks commitment (Problem 6, Application) and for
- promoting knowledge application.
- Data collecting by students on the basis of contemporary IT usage, systemizing and analyzing the results, making conclusions, finding evidence about the originality of the results, presenting and evaluating one’s own results and the results of the others.
The teachers are the mentors in these processes of a very high level of creativity.

**Third Stage. Student’s self research.**

The independent choice of self research is based on the high level of students’ tasks-commitment. The students search for information in big libraries and books or use didactical instruments developed by the training team. The main aim of the students is to express themselves as discoverers of geometric transformations, as solvers of mathematical problems and as creators of new geometric inequalities.

**The training of students from the control group (CG) includes:**

1. Presenting a geometric transformation and some of its applications.
2. Giving to students a task to create a transformation and new mathematical problems.
3. Presenting and evaluating the results.

**The criteria of the scientific research are:**

1. Degree of development of the Causal-Experimental Abilities (CEA).
2. Degree of Creativity (C).
3. Degree of development of Special Skills (SS).
4. Degree of development of Basic Skills (BS).
5. Degree of Tasks Commitments (motivation) (TC).
6. Degree of Knowledge Application (KA).
7. Qualities of the results: Theoretical development of a Geometric Transformation (TGT), Presentation (P), General Evaluation (GE).

**RESEARCH RESULTS**

The analysis of the reliability of research instruments, based on Kronbach’s coefficient α and the individual coefficients of each item, shows a very high level of reliability. Hence, the instruments are in accord with the goals of the empirical research and are appropriate for checking the correctness of the hypothesis raised.
The comparative analysis of the results of EG and CG is based on the Student and Fisher tests (for example, Table 1). The dynamic of the research characteristics development is based on dispersions (for example, Figure 1, Table 2) and Post-Host analysis (for example, Table 3).

**Table 1. Comparison of scopes of the EG and CG – Control experiment**

<table>
<thead>
<tr>
<th></th>
<th>Mean EG</th>
<th>Mean CG</th>
<th>t(28)</th>
<th>p</th>
<th>SD/EG</th>
<th>SD/CG</th>
<th>F(14,14)</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>CEA</td>
<td>24.667</td>
<td>10.400</td>
<td>12.379</td>
<td>0.000</td>
<td>3.132</td>
<td>3.180</td>
<td>1.031</td>
<td>0.955</td>
</tr>
<tr>
<td>C</td>
<td>25.733</td>
<td>13.533</td>
<td>7.287</td>
<td>0.000</td>
<td>4.267</td>
<td>4.882</td>
<td>1.309</td>
<td>0.621</td>
</tr>
<tr>
<td>SS</td>
<td>19.467</td>
<td>10.000</td>
<td>6.967</td>
<td>0.000</td>
<td>3.482</td>
<td>3.946</td>
<td>1.284</td>
<td>0.646</td>
</tr>
<tr>
<td>BS</td>
<td>21.533</td>
<td>9.333</td>
<td>12.055</td>
<td>0.000</td>
<td>2.696</td>
<td>2.845</td>
<td>1.114</td>
<td>0.643</td>
</tr>
<tr>
<td>KA</td>
<td>29.200</td>
<td>11.600</td>
<td>8.131</td>
<td>0.000</td>
<td>6.847</td>
<td>4.837</td>
<td>2.004</td>
<td>0.206</td>
</tr>
<tr>
<td>TC</td>
<td>9.267</td>
<td>4.800</td>
<td>7.007</td>
<td>0.000</td>
<td>1.438</td>
<td>2.007</td>
<td>1.949</td>
<td>0.224</td>
</tr>
<tr>
<td>TGT</td>
<td>12.733</td>
<td>6.000</td>
<td>7.995</td>
<td>0.000</td>
<td>2.017</td>
<td>2.563</td>
<td>1.616</td>
<td>0.380</td>
</tr>
<tr>
<td>P</td>
<td>12.333</td>
<td>7.467</td>
<td>5.270</td>
<td>0.000</td>
<td>2.193</td>
<td>2.825</td>
<td>1.659</td>
<td>0.354</td>
</tr>
<tr>
<td>GE</td>
<td>21.600</td>
<td>11.533</td>
<td>8.518</td>
<td>0.000</td>
<td>2.720</td>
<td>3.681</td>
<td>1.831</td>
<td>0.270</td>
</tr>
</tbody>
</table>

**Table 2. Results of dispersion analysis of the scale “CEA”**

<table>
<thead>
<tr>
<th>Factors</th>
<th>df</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group</td>
<td>(1,28)</td>
<td>68.041</td>
<td>0.000</td>
</tr>
<tr>
<td>Time</td>
<td>(2,56)</td>
<td>808.207</td>
<td>0.000</td>
</tr>
<tr>
<td>Group*Time</td>
<td>(2,56)</td>
<td>226.196</td>
<td>0.000</td>
</tr>
</tbody>
</table>
Promoting Creativity of Future Teachers In Mathematics

Figure 1. Dynamic of the Development of "CEA"

Table 3. Results of Post-Hoc analysis of the scale “CEA”

<table>
<thead>
<tr>
<th>Group</th>
<th>Experiment</th>
<th>t1</th>
<th>t2</th>
<th>t3</th>
<th>t4</th>
<th>t5</th>
<th>t6</th>
</tr>
</thead>
<tbody>
<tr>
<td>[1]</td>
<td>EE</td>
<td>0.00137</td>
<td>0.00137</td>
<td>0.399347</td>
<td>0.000143</td>
<td></td>
<td></td>
</tr>
<tr>
<td>[2]</td>
<td>GG</td>
<td>0.0000137</td>
<td>0.446931</td>
<td>0.0000158</td>
<td>0.000049</td>
<td>0.000143</td>
<td></td>
</tr>
<tr>
<td>[3]</td>
<td>GG</td>
<td>0.000137</td>
<td>0.446931</td>
<td>0.0000158</td>
<td>0.000049</td>
<td>0.000143</td>
<td></td>
</tr>
<tr>
<td>[4]</td>
<td>EE</td>
<td>0.399347</td>
<td>0.000158</td>
<td>0.000144</td>
<td>0.000143</td>
<td>0.000157</td>
<td></td>
</tr>
<tr>
<td>[5]</td>
<td>GG</td>
<td>0.000144</td>
<td>0.000144</td>
<td>0.000144</td>
<td>0.000143</td>
<td>0.000157</td>
<td></td>
</tr>
<tr>
<td>[6]</td>
<td>GG</td>
<td>0.000144</td>
<td>0.000144</td>
<td>0.000144</td>
<td>0.000143</td>
<td>0.000157</td>
<td></td>
</tr>
</tbody>
</table>

The establishing experiment shows that the EG and CG been appropriately chosen to the rules of the statistical theories and methods.

The general training motivates for self research more effectively the students from the EG than the students from the CG group because:

1. It offers easy and very attractive strategies for solving and creating mathematical problems based on the students’ skills and knowledge.
2. It ensures new opportunities for experimenting with one’s own ideas and creating new problems.
3. It forms students’ self-confidence.
The lecture as a method of training in the control group is a synthesis of many original scientific results but it does not involve students deeply into the process of discovering new knowledge. For this reason, analogy is a general characteristic of the learning students’ style. The students follow the standard schemes presented by the teacher:

1. The students try to formulate a theorem for the existence of a transformation without the full combination of all factors and without any deep analysis.

2. They don’t use flexible searching of appropriate mathematical methods and because of that they don’t rediscover many transformation characteristics.

3. They don’t create new problems or give original solutions.

The students from the EG use the didactical instrument (Application) for discovering “Inversion” as a transformation and its characteristics. The students use them for creating effective solutions to geometric inequalities on the basis of applying different problem-solving strategies or mathematical methods for simplicity of the initial inequalities, combining transformation characteristics, using generalizing inequalities, analyzing the results, etc. The students apply different strategies and IT in searching, processing and presenting the information. They exchange ideas and experience.

The optimal level of the motivation is the beginning of the third stage – self research of the student from the EG. The students from the CG show interest in the transformation method but they need a task from a teacher to start their research as a home assignment.

The level of development of the dynamic characteristics measured in the control experiment through the created and presented students’ products is higher in the EG than in the CG (Table 1, Figure 1).

The control experiment shows also the higher level of the values of all measured characteristics in the EC with respect to the CG (Table 1).

The statistical analysis proves that the raised research hypothesis is correct. Thus, applying the conceptual training model in the area of the transformation method sets the pattern for the developments of the productive potential of students and the creative character of the results from their activity.

The characteristics of the created by the students’ transformations, their applications and multimedia usage for results presenting, as well as their overall assessment from an academic and feasibility point of view are defined by three essential parameters: theoretical development of a geometric transformation, presentation, general evaluation.
The students from the EG discover transformations by combinations of different elements from the didactic instruments or by geometric constructions. They raise and prove the correctness of the hypothesis, discover transformation characteristics, analyze the results, make transformation models, create new problems, search information about interesting inequalities or authors. There aren’t any limitations to discovering all elements and characteristics of the transformations because it is a very difficult task.

The students from the CG don’t use all possibilities given by the didactical instruments. They don’t discover many characteristics of the transformation, don’t use geometric interpretations. A small number of the students create problems which are not so complex or new.

The presentations of the students from the EG are of a higher level than those from the CG.

The products of the students from the EG have a creative character. They reflect the student progress in learning and creating new scientific knowledge in the area of geometric transformations.

SOME INTERESTING PROBLEMS
OF THE STUDENTS FROM THE EG.

Problem.

Let $\Delta \left( A_1A_2A_3 \right) \,$ is a triangle with an interior points $P$. Let $r_1, r_2, r_3$ and $R_1, R_2, R_3$ are the distances from $P$ to $A_2A_3, A_3A_1, A_1A_2$, and to $A_1, A_2, A_3$, respectively. The points $P$ and $P_1$ are isogonally conjugated. In any triangle $\Delta \left( A_1A_2A_3 \right) \,$, the following inequalities hold:

1. $\frac{r_1R_1}{r_1r_2 + r_1R_1} + \frac{r_2R_2}{r_2r_3 + r_2R_2} + \frac{r_3R_3}{r_3r_1 + r_3R_3} \geq 2$, 
2. $r_1R_1 + r_2R_2 + r_3R_3 \geq 2 (r_2r_3 + r_2r_1 + r_3r_2)$, 
3. $R_1R_2R_3 \geq \left( r_2 + r_3 \right) \left( r_3 + r_1 \right) \left( r_1 + r_2 \right)$. 

CMEG-5 253
Problem.

For any positive numbers \( x, y, z \), the following inequality holds:

\[
\sum \sqrt{(z + x)(x + y)} \leq 2(x + y + z).
\]

Problem.

In any triangle with sides \( a, b, c \) and area \( F \), the following asymmetric inequality holds:

\[
9a^2 - 3b^2 + 5c^2 \geq 4\sqrt{3}F.
\]

The conclusions are that the conceptual training model, based on IT, provides conditions for the development of the research potential of the gifted students from the EG to a much larger extent and faster than the “standard” one’s, applied in the CG and for the creating new scientific knowledge with high quantities by students from the EG.

The factor analysis and correlation coefficients show that high degree of development of the students’ research potential in the experimental group is determined by application of conceptual training model and determinates the high level of qualities of the students’ results. Thus, the raised research hypothesis is correct.

The linear regression analysis presents that the main characteristics important for the high level of students’ results are the degree of development of the causal-experimental abilities and creativity.

The quality and qualitative analysis leads to the correctness of the research hypothesis.

CONCLUSIONS

The research on promoting creativity of university students is an important part in finding new ways for training of “creative-productive gifted” students, developing their potential and preparing them for future successful scientific research or long life learning that will be benefit for our society.

The using of methods of instructions included into the conceptual research model provides conditions for promoting of students’ creativity by:
Promoting Creativity of Future Teachers in Mathematics

1. Development of students’ knowledge, creative abilities and research and IT usage skills to a much higher level than the “standard” extracurricular activities in the field of mathematics.

2. Development of causal-experimental thinking of students which is basis for creating new mathematical knowledge in mathematical areas with high level of complexity.


4. Development of a high level of motivation for self research.

5. Application the student potential in interesting scientific area based on IT and Computer Algebra System usage.

REFERENCES


APPLICATION: EXAMPLE OF A DIDACTICAL INSTRUMENT

Problem 1.

Let there are given:

\[ \prod R_i \geq \prod (r_i + r_i) \rightarrow (\prod R_i)^2 \geq \prod (r_i R_i + r_i R_i) , \]

\[ 3 \sum a_i^2 \geq \sum a_i^2 \rightarrow 3 \sum (r_i R_i)^2 \geq \sum (a_i R_i)^2 , \]

\[ \sum a_i R_i \geq 2 \sum a_i r_i \rightarrow \sum R_i R_i R_i \geq 2 \sum a_i r_i R_i^2 , \]

\[ \sum a_i^2 \geq 4 \sqrt{3} \rightarrow \sum a_i (p - a_i) \geq 2 \sqrt{3} , \]

\[ \sum R_i R_i \geq 4 \sum \ell_i \ell_i \rightarrow \sum R_i \geq 4 \sum \ell_i / R_i . \]

1. Find all combinations of the transformation elements.
2. Discover appropriate combination of triangle elements and format a theorem for existing of a transformation.
3. Prove the theorem.
4. Format a consequence and definition of the transformation.

Problem 2.

Prove the inequalities: 

\[ \sum \frac{1}{r_i r_j} \geq 2 \sum \frac{1}{r_i R_i} , \quad \sum R_i R_i R_i \geq 2 \sum \ell_i / R_i . \]

\[ K^2 \geq \prod (\ell_i R_i + \ell_i R_i) , \quad 2 \sum R_i \geq \sum \ell_i . \]

Apply the new transformation two times to these inequalities. Make conclusions. Define the elements of the triangle.
Δ'' (dual of Δ' with respect to the transformation) as functions of the elements of Δ.

Problem 3.

Find many new inequalities on the basis of Klamkin's inequality
\[ \sum x_i \sum x_i R_i^2 \geq \sum \alpha_i x_i x_j, \]
where \( x_1, x_2, x_3 \) are real variables and the equality occurs if and only if
\[ \frac{a_1 r_1}{x_1} = \frac{a_2 r_2}{x_2} = \frac{a_3 r_3}{x_3}. \]

Problem 4.

Find applications of the transformation in the books or big electronic libraries.

Problem 5.

Create new geometrical inequalities with the help of the transformation.

Problem 6.

Find which inequalities are original and which are not. Cite the sources.

Problem 7.

Apply the transformation for:
\[ \sum \frac{a_1}{r_1}, \frac{r_2 r_3}{R_1 R_2 R_3} = \frac{k}{K}, \frac{1}{2kR} \sum \alpha_i R_i^2, \] \[ K \geq 8k. \]

Make conclusions.

Problem 8.

Apply composition of the transformation and MDT for the inequality
\[ 7 \sum m_i \prod m_i + 9S^2 \leq \frac{3\sqrt{5}}{2} (\sum m_i)^2. \]
THE USE OF COMPONENTS OF MATHEMATICAL ABILITIES FOR INITIAL IDENTIFICATION OF MATHEMATICALLY PROMISING STUDENTS

TETYANA VILKOMIR AND JOHN O’DONOGHUE

Department of Mathematics and Statistics, University of Limerick, Ireland

Abstracts

This study investigates the possibility of the application of Krutetskii’s approach to mathematical giftedness to the practical identification of mathematically promising students. Sets of problems for testing such components of mathematical abilities as flexibility, reversibility, and ability to generalize were developed by the authors. These sets could be used by researchers and teachers for initial identification purposes.

INTRODUCTION

The purpose of this paper is to present authors’ ideas for practical applications of Krutetskii’s (1976) work to the problem of identifying mathematically promising students. During the last thirty years, significant projects on identification and development of mathematically gifted students were conducted at the Johns Hopkins University in Baltimore (Benbow and Stanley, 1983) and the University of Hamburg (Wagner and Zimmermann, 1986). However, these researchers mostly focused on finding highly mathematically gifted students. Wagner and Zimmermann (1986, p.243) stressed that “Any concept for identifying and fostering mathematically gifted students necessarily depends on specific administrative determinants (e.g., school system of a country) and a “system of values” (e.g., educational objectives)”. The main focus of our research is students that were identified by Krutetskii as mathematically able, above average or capable. We attempt to develop identification procedures that could be used in Ireland for 12-13 year old students. In our work we adopted the term mathematically promising students that was introduced by the Task Force on Mathematically Promising Students (Sheffield, 1999). In our opinion, the term mathematically promising much better reflects the fact that these students have the potential to succeed in becoming highly mathematically gifted than the terms
mathematically able, above average or capable used by Krutetskii. Their
description in this way emphasizes the main goal of identifying such students. This
goal is to find students who are in need of special attention and support for
developing their inborn mathematical abilities not because these students are
incapable but because they are more capable than others. At the same time, this
description gives us an opportunity to include both, students of above average
mathematical ability and highly mathematically gifted students, in the process of
identification.

HYPOTHESIS

In his work, Krutetskii (1976) stressed that abilities like reversibility, flexibility,
and ability to generalise would be the basis for identifying mathematically able
students because they appeared to develop naturally in such students. However,
students of average mathematical ability did lack these abilities and students of low
mathematical ability did not show them at all. Later researchers (Rachlin, 2000;
Sheffield, 1999; Williams, 2002) supported this approach for identifying
mathematically gifted students. Thus, Rachlin (2000, p.4) identified these three
basic processes as starting points for the successful study of mathematics. Although Keisswetter (1985) formulated his own definition of mathematical
giftedness, he emphasized the following mathematical activities:

- recognising patterns and rules and finding related problems – corresponds
with Krutetskii’s ability to generalise;
- changing the representation of the problem and recognizing patterns and rules
in this new area – corresponds with Krutetskii’s flexibility of mental
processes;
- reversing process – corresponds with Krutetskii’s reversibility of mental
processes.

At the same time, if student’s reversibility, flexibility, and ability to generalise are
well developed then we can argue that there is a high possibility for the rest of
components being also well developed. For example, we believe that the ability to
generalise is associated with the ability for formalised perception of mathematical
material and broad generalisation would definitely require the ability for logical
thought or what Keisswetter (1985, p 301) called “comprehending very complex
structures and working within these structures”. Flexibility of mental processes in
mathematical activity and reversibility of mental processes in our opinion are
associated with striving for clarity, simplicity, economy, and rationality of
solutions.
As a first step, we have developed a table that allows us to ascertain the level of development of components of mathematical ability for each student (Vilkomir, 2005; Vilkomir, O’Donoghue and Murphy, 2006). The table provides signs of development of each component of mathematical abilities and shows relationships between components of mathematical abilities and mathematical promise. As the next step we designed sets of problems to test the level of development of the above named three components of mathematical ability (different sets for different components). Such sets would be used in combination with the table. The signs of development of each component of mathematical abilities are possible to observe during problem solving by a student (Krutetskii. 1976). We attempted to design sets of problems in a way that written solutions would give a clear picture of the level of development of components without researchers actually observing students as they do problem sets. We think that if students provide full solutions for each problem then researchers guided by the designed table would be able to ascertain the level of development of these components of mathematical ability for each student. This would enable researchers to identify mathematically promising students.

SPECIFICATIONS FOR SETS OF PROBLEMS

We developed three sets of problems – one for each of the following components of mathematical ability: flexibility, reversibility, and generalisation. In order to effect the design of problems we needed to answer several questions:

- How to design sets of problems in a way that they would give indicators of mathematical promise without a teacher actually observing the process of solving?
- How to avoid accidental correct solutions?
- How to organise problems that are designed to test this component and ability to generalise so that they would be solved in a very particular order?
- To design sets of problems for initial identification of mathematically promising 12-13 year old students. Therefore the level of difficulty of designed problems should be kept on the level that would not require knowledge beyond the school curricula in Ireland. At the same time the problems should not be too easy or obvious for students. How to make sure of this?

To provide researchers with indicators of mathematical abilities and to avoid accidental correct answers students are asked for full solutions. Moreover, for problems designed to test flexibility of mental processes students are asked to provide as many different solutions for each problem as they can think of.
future, to save teachers time analysing the results designers would provide teachers with sets of possible solutions for each problem (as many ways as possible).

Table 1

<table>
<thead>
<tr>
<th>Section I. Problems with seemingly unimportant differences.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Problem 1</strong> Solve the following two problems in the order in which they are presented.</td>
</tr>
<tr>
<td>a) Two cars leave from different petrol stations on the same road at the same time heading towards each other. One travels at 29 km/h and the other at 34 km/h. If they pass each other after 5 hours, what is the distance between the two petrol stations?</td>
</tr>
<tr>
<td>b) Two cars leave from different petrol stations on the same road at the same time heading in the same direction. One travels at 29 km/h and the other at 34 km/h. If they pass each other after 5 hours, what is the distance between the two petrol stations?</td>
</tr>
<tr>
<td><strong>Problem 2</strong> Tourists take a boat trip involving 4 hours on a lake followed by 5 hours on a river. If the engine speed of the boat is 24 km/h, the lake has no current, and the speed of the river current is 3 km/h, what distance did the tourists travel? Consider all cases. (The boat does not change direction during the trip on the river).</td>
</tr>
<tr>
<td><strong>Problem 3</strong> Solve the following two problems in the order in which they are presented.</td>
</tr>
<tr>
<td>a) The distance between two cities is the sum of 93 km and half of this part of the distance. What is the distance between the cities?</td>
</tr>
<tr>
<td>b) The distance between two cities is the sum of 93 km and half of the total distance between the two of them. What is the distance between the cities?</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Section II. Problems with several solutions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Problem 4</strong> The following problems have several ways of solving them. For each of the following problems find as many different ways of solving the problem as you can.</td>
</tr>
<tr>
<td>a) A farmer lives 24 km from the nearest village. He cycled from his home to the village at 12km per hour, and it took him 3 hours to return home from the village. On which stage of the journey, to the village or back, did he spend more time?</td>
</tr>
<tr>
<td>b) There are 160 books on one shelf and 200 books on the other. How many books should a librarian move from the second shelf to the first shelf so that both shelves contain the same number of books?</td>
</tr>
<tr>
<td>c) Peter and Jesse took 18 carrots to the Zoo. They fed ponies and donkeys, 5 animals altogether. If each pony ate 4 carrots and each donkey ate 3 carrots, how many donkeys and how many ponies did they feed?</td>
</tr>
<tr>
<td>d) The distance between two cities is 816 km. A passenger train and a freight train depart from these cities at the same time going towards each other. The speed of the passenger train is 6 km per hour greater than the speed of the freight train. They meet after 8 hours. What is the speed of each train?</td>
</tr>
</tbody>
</table>
Addressing the third question, we put each problem on a separate page with a clear instruction when and where to go from this page.

Before being presented to students designed problems were approved as fit for purpose and suitable for the age group by the panel that consisted of three mathematicians and three experienced mathematics teachers. In our opinion, this assessment helped us to keep problems at suitable level of difficulty.

As an example we provide the set of problems that we used to analyse students’ flexibility of mental processes.

**Problems to test Flexibility of mental processes**

These problems are designed to test the students’ ability to switch from one mental operation to another. Problems in the first section are designed to assess the ability of students to see that these seemingly very similar problems require very different solutions and how well students can switch from one solution to another. We believe that students of average or below average mathematical ability would experience difficulties in distinguishing between a) and b) in Problem 1. Students of above average mathematical ability would “see” some kind of a mental picture for each of these two problems. At this point we would like to stress that “mental picture” in this context does not necessarily mean visual picture but deep understanding of interrelations between components of the given problem. When a mathematically promising student sees a problem of the same mathematical type next time he/she would recognise this even if mathematically unessential components are absolutely different (for example, if they have problems about cyclists, boats, or trains instead of cars). Similarly, promising students would “see” two different problems in Problem 2: first one - when the boat goes with the current on the river; and another one – when the boat goes in the opposite direction. Average or below average students would have difficulty recognising two problems. Most probably they would be satisfied with the first situation they could find.

Problems in the second section are designed to test if students can see that the given problems can be solved in different ways, and how well students can move from one method of solution to another. In this way it is possible to see how flexible students’ mental processes are - meaning the level of their “ability for a rapid reconstruction of mental activity, for “breaking” a just-established solution pattern and replacing it with a new one” (Krutetskii, p. 277). In this set of problems, it is important to see not only answers but full solutions. The more different solutions students can find the more flexible their mental processes are.
CONCLUSION

The authors have described their attempt to develop a practical application of Krutetskii’s approach to the important problem of identifying mathematically gifted students. Examples of problems that were to identify flexibility of mental processes are presented. These problems are meant to be used in conjunction with a practical guide for determining the level of development of components of mathematical abilities in individual students, in terms of specified observables, as presented in a set of structured reference tables (Vilkomir, O’Donoghue, and Murphy, 2006). A significant motivation for this work is the desire to avoid time consuming and resource intensive practices such as interviews, special examinations and summer schools which heretofore have been used successfully because these practices are now out of reach for all but very wealthy countries or highly ideologically driven systems.

References


