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THINKING PROCESSES OF GCM: A COGNITIVE INQUIRY

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Abstract

Research concerning the cognitive processes of human individuals identified as Gifted Children in Mathematics (GCM) is scant. This project proposes a research into such cognitive processes. The leading theoretical perspective is APOS, a theory that has already proven its effectiveness in identifying the mental structures individuals construct and use in learning mathematical concepts. This research dealt with ordinary students of different age levels, and a variety of mathematical concepts. We propose to investigate the question whether GCM develop similar mental structures when they learn mathematical concepts, only quicker and in more efficient ways, or do they use completely different ways? The mathematical concepts that we will investigate are infinity and function. Answers to our research question might on the one hand enhance better identification methods of GCM, especially in underrepresented populations, and on the other hand might improve our understanding of how ordinary learners learn mathematics. In addition, our investigations could point to pedagogical strategies for better helping GCM realize the promise of their special gifts.

PROJECT DESCRIPTION

The purpose of this project is to investigate the thinking processes of Gifted Children in Mathematics (GCM). We will use APOS Theory, a well-tested theory of learning mathematics, to describe the thinking of GCM and to determine if the cognition of these individuals involves thinking processes that go beyond those possessed by non-GCM. We ask if APOS Theory, a well-tested theory of learning mathematics, describes the thinking of GCM or if the cognition of these individuals involves thinking processes that go beyond those possessed by non-GCM. We focus on the mathematical concept of infinity for several reasons. This is a concept that pervades mathematics and is critical for deep understanding of most topics in advanced mathematics; aspects of the concept of infinity can be accessible to sufficiently bright students at any grade level; and our group has experience in cognitive analyses of this concept, as well as in using our theoretical results to design and implement effective pedagogy. As a secondary focus, we will also consider, for similar reasons, the mathematical concept of function.

Most of the members of the proposed project have been involved for the past 14 years in a group known as Research in Undergraduate Mathematics Education Community (RUMEC).

The main work of RUMEC has consisted of collaborative research and curriculum development in mathematics education, generally using APOS Theory. This theory is based on the ideas of J. Piaget, reinterpreted to be applicable to mathematics at the secondary and post-secondary levels. Its main hypothesis is that an individual must apply certain mental mechanisms to build specific mental structures that are used in learning a mathematical concept. The pedagogy is aimed at helping students construct the mental structures necessary for the development of each of these concepts, as predicted by APOS. For further description of the theory, see Asiala et al. (1996) and Dubinsky and McDonald (2000).

For a wide variety of mathematical concepts (e.g., infinity, functions, sets, propositional and predicate calculus, mathematical induction, limits, calculus, abstract and linear algebra) qualitative and quantitative studies have led to the identification of specific mental mechanisms and structures which has led to effective pedagogy. Details of these studies and their results, in terms of student learning, have been reported in numerous publications. In the presentation I will elaborate about how APOS describes the development of mathematical concepts in an individuals' mind. In particular, I will present results of research conducted by members of our group, describing the development of the concepts infinity and function according to APOS. I will describe APOS as a paradigm for research in Mathematics Education, and pedagogical implications of APOS, as related to the concepts we are concentrating on.

We are an international group and of the seven members of the proposed project, 5 live in the U.S., one in Mexico and one in Israel. Thus we will work in all three countries. Our subjects will come from all grade levels and possibly some undergraduates. We focus our attention initially on those children identified as gifted in mathematics, and as we develop what may be new and deeper understandings of the thinking processes of GCM we will study other children who may not yet be identified as gifted but do exhibit substantial portions of the thinking of GCM. In particular we will study children from underserved populations (African-American, Hispanic, Native).

Our idea is to use the APOS theoretical framework and Game Playing strategies to study the thinking of GCM. This work will be important first of all because of the critical role of mathematics in all aspects of our culture and the strong need for major increases in the number of individuals in our society who are capable of high-level mathematical thinking. A better understanding of how the thinking of GCM compares to that of other children would provide valuable insights that could help teachers and possibly inform public policy. Such an understanding would raise, and begin to answer, vital questions about Giftedness. For instance, if we were to find that GCM make different kinds of mental structures or higher quality mental structures, or construct mental structures with greater speed, what would be the implications for the teaching of mathematics? What would be the implications for mixing non-GCM with GCM? How would this inform our thinking about current approaches to the mathematical education of GCM?

Our studies may also raise questions about the identification of GCM. Current methods involving tests and grades are highly reliable in that those identified as GCM turn out to be very successful in their learning (see Lubinski & Benbow, 2006). But is it possible that standard measures of intelligence and identification of GCM are missing students from low socioeconomic or non-Caucasian ethnic backgrounds? Could the kinds of cognitive studies that have been successful in pointing to effective innovative pedagogies also provide alternative tools to identify mathematical giftedness and incorporate in our society what has been called “underexploited resources”?

The main outcome of our project would be a deeper understanding of GCM and this could lead to improved pedagogical strategies that would help these children better develop the promise of their gifts. It would also help ensure that socio-economic barriers for GCM in underrepresented populations are lowered. Finally, any investigations of the type we are proposing have the potential for providing insights that could be applied to the mathematical education of all children.

The enduring impact of our project will lie in the increased mathematical power of the most intelligent members of our society and in the contributions they will make to science and industry throughout their lives. Enhancing the learning of those identified as GCM and developing methods to see that even more children should be so identified would increase the number and quality of those working at the highest levels of their professions and have a salutary effect not only on their contributions but on the work of those in contact with them. The nature of our society in this century demands a higher level of mathematical thinking by more people. The development of GCM, their work and their effect on others, will lead to major progress in meeting this need.

STAGES OF THE PROJECT

The first phase of the project is planned for two years, as described below:

The first year of the project is dedicated to the production of a summary of the literature on GCM; developing contacts with existing workers and institutes concerned with GCM, developing instruments for investigation (questionnaires and interview protocols, including teaching interviews), making arrangements for the subjects of the study, and conducting pilot studies to validate our instruments.

In the summer after the first year, we will revise our instruments based on the pilots and make final arrangements for the main studies to take place in the second year.

We will conduct our studies in the second year, transcribe the recordings of interviews, translate where necessary (from Hebrew and Spanish to English) and analyze our data.

In the summer of the second year, we will write reports and research papers based on our studies and plan follow-up activities in what we hope will be a continuing project. Future work will expand on the studies described here and investigate the relationship between existing programs for GCM and what we have learned. If warranted, the project will work on the development of alternative approaches to the education of GCM.

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THE RELATIONSHIP BETWEEN MATHEMATICS ABILITY AND MATHEMATICS ANXIETY IN GIFTED STUDENTS

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ABSTRACT

In this study the relationship between mathematics ability and mathematics anxiety of gifted students were examined. The sample of study consisted of 122 students (50 gifted, 72 normal) from Beyazit Ford Otosan Primary School in Istanbul. The research data were derived from "School and College Ability Test (SCAT)" and "Math Anxiety Scale". Demographical information form, which was prepared by the author of this paper, was used. A T-test and Pearson Correlation Analysis were used for statistical analyses. In results correlation wasn't found between math ability and math anxiety.

Key words: Mathematics anxiety, Mathematics ability, gifted students.

INTRODUCTION

To help us understand anxiety, we will first describe a related emotion fear. Fear is an emotion produced by danger. As the danger increases, so does the fear. Taking a driving test, interviewing for a job, or walking down an unlighted street at night may produce some fear, but it will be easily controlled. As the danger increases, however, symptoms such as pounding of the heart, a sinking feeling in the stomach, shaking and trembling and cold sweat become more difficult to control. In extreme fear, or panic, thought deteriorates into a nightmare of distorted mental images, and the person may run widely about, crying, shouting, and laughing in rapid succession. Though you have probably experienced panic, you may not recall just how excruciatingly unpleasant it was because it is usually forgotten soon after the crisis is over (Dustin, 1969).

Anxiety has been widely recognized as a fundamental emotion and a basic condition of human existence by the behavioral scientists. Freud, being the first to attempt to investigate anxiety, emphasized the role of impulses in the development of anxiety. According to Freud, anxiety is a signal indicating the presence of a danger situation. If the source of danger is from the external world, anxiety is objective. If the source stems from internal impulses, then anxiety is neurotic (Spielberger, 1966).

Test –Taking anxiety is the feeling of fear you have when your performance is about to be evaluated. When you experience this type of anxiety, your abilities to think and to pay attention plummet, right at a time when you need them most (Johnson, 1997).

Mathematics anxiety and its correlates have been studied for many years. According Tobias and Weissbrod (1980) “The term was used to describe the panic, helplessness, paralysis, and mental disorganization” observed in some people required to solve mathematical problems (Tobias, 1980).

Mathematical talent refers to an unusually high ability to understand mathematical ideas and to reason mathematically, rather than just a high ability to do arithmetic computations or get top grades in mathematics. When considering mathematical talent, many people place too much emphasis on computational skill or high ability in replicating taught mathematical procedures. Unless mathematical talent is correctly perceived, however, important clues can be overlooked and less important clues can be given too much significance (Miller, 1990).

In this study, it is aimed to show relationship between mathematics anxiety and mathematics ability in gifted and normal students. Mathematic anxiety, affect mathematical achievement negatively, in math lectures. As mathematics anxiety increase, mathematics achievement decreases. Mathematics achievement is negatively affected by mathematics anxiety. As this result, in this study, it is aimed to indicate correlation between mathematics ability and mathematics anxiety.

MATHEMATICS ANXIETY

About twenty-five years ago phenomenon of “Math Anxiety” was identified and described – by well-known people, educators endeavouring to explain why some people have more trouble learning mathematics than others. We don’t hear much about math anxiety in math departments because such departments are full of people who don’t have it. Math anxiety is an inability by an otherwise intelligent person to cope with quantification and, more generally, with mathematics. Frequently the outward symptoms of math anxiety are physiological rather than physiological. When Confronted by a math problem, the sufferer has sweaty palms, is nauseous, has heart palpitations, and experiences paralysis of thought. Oft-cited examples of math anxiety are the successful business person who cannot calculate a tip, or the brilliant musician who cannot balance a check-book. This quick description does not begin to describe the torment that those suffering from math anxiety actually experience (Krantz, 2000).

In today's world, it is essential to educate students to build confidence in mathematics learning since both academic and daily lives require strong mathematical knowledge and skills. However, things such as irrelevancy to everyday life and requirement of high intelligence attributed to mathematics science may cause anxiety (Richardson, 1980).

Most of researchers reported that a general agreement that levels of math anxiety negatively affected academic performance in mathematics. Mathematics anxiety contributes to mathematics avoidance and poor mathematics performance has been particularly emphasized for girls. But the changing role of girls in the society, mathematics anxiety is slowly becoming to direction of equal opportunity (Betz, 1978).

MATHEMATICS ABILITY

Attempts of this kind have been made more than once, but hitherto there has been no fixed definition of mathematical ability that would satisfy everyone. Perhaps the only thing about which all investigators agree is that one should distinguish between ordinary, "school" ability for mastering mathematical information, reproducing it, and using it independently and creative mathematical ability, related to the independent creation of an original product that has a social value (Krutetskii, 1976).

Most foreign psychologists who have "school" ability in mind tend to understand it as an ability to do mathematical tests or problems. Of course, such a definition lacks content and needs to be made concrete. Some psychologists have noted certain fundamental properties of the mind, which they believe underlie a scholastic ability for mathematics (Krutetskii, 1976)s.

As early as 1918, in a work by A.L. Rogers (1918, Krutetskii, 1976) two aspects of mathematical ability were distinguished: the reproductive (related to the function of memory) and the productive (related to the function of thought). Krutetskii (1976) constructed a well-known system of mathematical tests. W. Betz (1923, Krutetskii, 1976) defined mathematical ability as a ability to have a clear awareness of the internal connections in mathematical relations and to think precisely with mathematical concepts. A. Wenzl (1934, Krutetskii, 1976) defined it as the ability to establish meaningful connections in mathematical material. A. M. Blackwell (1940, Krutetskii, 1976), who published his factorial study of mathematical ability in 1940, indicates that these abilities can be interpreted as abilities for selective thinking in the realm of quantitative relationships (quantitative thinking) and for deductive reasoning, and as the ability to apply general principles to particular cases in the realm of numbers, symbols, and geometric forms. W. Lietzmann (1941, Krutetskii, 1976) notes the ability to reason in particular situation with thee use of symbols from a mathematical language. The Finnish psychologist R. Meinander (1958, Krutetskii, 1976) speaks of mathematical ability as a complex quality including intelligence, memory, interest, and volitional factors. This is a new statement of the matter, which is related to a broad personal understanding of ability.

In the twenties several introspective works on the structure of mathematical giftedness also appeared. Annie E. Cameron (1925, Krutetskii, 1976) selected the following factors (components) : (1) an ability to analyze a mathematical structure and to recombine its elements, (2) an ability to compare and classify numerical and spatial data, (3) an ability to apply general principles and to operate with abstract quantities, and (4) the power of imagination. V. Kommerell's work (1928, Krutetskii, 1976), lists such factors as : (1) clear logical thinking, proper use of logical methods; (2) the power of abstraction; (3) combinatorial ability; (4) an ability for spatial conception and for operating with spatial forms; (5) critical thinking, the ability to abandon an erroneous train of thought; and (6) memory. In his work "Die mathematische Beganbung", H. Thomas (1929, Krutetskii, 1976) differentiated such components as: (1) an ability for abstraction, (2) an ability for logical reasoning, (3) specific perception, (4) the power of intuition, (5) an ability to use formulas, and (6) mathematical imagination. Thomas also noted the importance of a distinctive "automatization" of reasoning and of operations with numbers.

METHOD

Sample

This study has been conducted with 122 students in Ford Otosan Beyazit Primary School. 56 (%45,9) girl, 66 (%54,1) boys. 50 (%41,0) of the sample students have been defined as gifted, 72 (%59,0) of the sample students have been defined as normal students. The distribution of the students according to their classes is 57 (%46,7) fourth classroom, 65 (%53,3) fifth classroom.

Measures

Mathematics Anxiety was assessed with Math Anxiety Scale (Erol, 1989). It contains 45 items and it 4 dimensions: mathematics exam anxiety, anxiety in math lesson, mathematics anxiety in daily life, self-confidence in mathematics. Also the questionnaire was used for demographic characteristics.

Mathematics Ability was assessed with The School and College Ability Tests (SCAT). SCAT Series III were developed in 1980 by Educational Testing Service (ETS). All rights to SCAT Series II and Series III were purchased from ETS in 1996 by The John Hopkins University Center for Talented Youth (CTY). At that time, the SCAT was revised to eliminate or change any items that contained gender, ethic, or culturally biased wording. CTY has used the SCAT since 1985 to identify highly able students who may need special education programs such as summer courses or distance learning opportunities, as well as to plan appropriate education plans for individual students. CTY's Elementary Students Talent Search established in 1997 uses newly constructed and computer-administered forms of the SCAT to identify academically talented students nationwide.

The SCAT measures verbal and quantitative abilities of students in grade 3-12. It is useful in comparing students or classes, comparing performance on the verbal and quantitative parts, estimating growth in abilities over time, and predicting success in related achievement areas. SCAT tests measure a student's aptitude for learning rather specific skills or content taught in specific grades. To measure a student's specific skill levels or content knowledge, an achievement test should be used.

Statistical Procedures

Correlation Coefficient has been estimated to determine the relation between Math Anxiety Scale and Math grades. To investigate the relationships among gender, intelligence level and class, independent group T-Test has been applied. Also Pearson Correlation Analysis has been estimated to determine the relation between Math Anxiety Scale and Math Ability Test.

FINDINGS AND DISCUSSION

The relationship between sub-dimensions of Math Anxiety Scale and total score, math anxiety and math ability, math anxiety and other variables, math ability and other variables were investigated. Some of the tables are given below.

As seen from the table 1, no significant difference between male and female students' mathematical average has been spotted according to the results of t-test which has been applied to determine the mathematical average of total match anxiety score and sex variable. Preston (1987) has also founded similar results. However, Erktin, Dönmez and Özel's (2006) research shows that math anxiety differentiates with respect to gender. According to this study, girls were more anxious the boys. This situation might the resulted in different age groups of sampling in these two studies (Batdal, 2007).

Table 1.The results of independent t-Test to determine the differentiation rate of Total Math Anxiety Score according to gender variable

Score	Gender	N	$\overline{x}$	ss	$Sh_{\overline{x}}$	t **Test**		
						Sd	t	p
Total Math Anxiety	Boy	66	74,0152	19,74880	2,43091	120	1,057	0,293
	Girl	56	70,5714	15,52108	2,07409			

p>.05

Table 2. Results of the independent Group t Test conducted to determine as to whether or not the School and College Ability Test (SCAT) Scores are differentiating depending on the Gender variable.

Score	Gender	N	$\overline{x}$	ss	$Sh_{\overline{x}}$	t **Test**		
						Sd	t	p
SCAT	Boy	66	33,5455	10,09521	1,05704	120	1,840	0,068
	Girl	56	36,5455	7,91013	1,05704			

p>.05

The above table indicates that the independent group t test that was applied to determine as to whether or not the arithmetic averages of the school and college ability scores would demonstrate a meaningful difference as per the gender variable yielded a statistically meaningful difference between the arithmetic average scores achieved by male and female students. The school and College Ability Test scores of the male students were lower than those of the female to a meaningful extent. This may result from a lower anxiety level of female students during the math class. Thus, they may be more successful at mathematics. Although not fully correlated with ability, this may have an indirect correlation.

Table 3. The results of Pearson correlation analysis to determine the differentiation rate of Total Math Anxiety Score according to mathematics scores

Variable	*N*	*r*	*p*
Math Anxiety Math Score	122	-0,220	0,015

As a result of the research of Pearson Correlation Analysis which have been done to determine the relationship between the gained scores in the table and the math marks, a meaningful statistical ($p<.05$) relationship going to has been spotted ($r=-.220$; $p<.05$). This result shows us that mathematic anxiety, affect mathematical achievement negatively, in math lectures. As mathematics anxiety increase, mathematics achievement decreases. Mathematics achievement is negatively affected by mathematics anxiety (Betz, 1978, Suinn, Taylor, Edwards, 1988, Erol, 1989, Erktin, Dönmez, Özel, 2006, Batdal, 2007).

Table 4. The results of independent t-Test to determine the differentiation rate of Total Math Exam Anxiety Score according to intelligence variable

Score	Intelligence	N	$\overline{x}$	ss	$Sh_{\overline{x}}$	t **Test**		
						Sd	t	p
Math Exam Anxiety	Gifted	50	28,2000	8,43946	1,19352	120	1,080	0,282
	Normal	72	29,9861	9,34095	1,10084			

p>.05

As seen from the table, no significant difference between gifted and normal students' mathematical average has been spotted according to the results of t-test which has been applied to determine the mathematical average of math exam anxiety score and intelligence variable (Batdal, 2007).

Table 5 bellow indicates that the independent group t test that was applied to determine as to whether or not the arithmetic averages of the school and college ability scores would demonstrate a meaningful difference as per the intelligence variable yielded a statistically meaningful difference between the arithmetic average scores achieved by students with normal and superior intelligence. The statistical School and College Ability Test scores of students with superior intelligence were meaningfully different than those with normal intelligence level. The school and college ability scores of the superior intelligent students had a meaningfully higher level than those with normal intelligence. This may result from the fact that students with superior intelligence have a higher mathematical ability.

Table 5. The results of the independent Group t Test applied to determine as to whether or not the School and College Ability Test differentiate accordşng to the Intelligence Variable

Score	Intelligence	N	$\overline{x}$	*ss*	$Sh_{\overline{x}}$	t Test		
						Sd	t	p
SCAT	Gifted	50	41,4600	5,87700	0,83113	120	7,983	.000
	Normal	72	30,4306	8,44840	0,99565			

p>.05

Table 6. Regression Analysis Results on the Prediction of the SCAT Competence according to the Anxiety Capability

Variable	B	Standart Error β	β	T	p
Constant	29,108	5,206		5,592	.000
Anxiety	.070	.062	.103	1,137	.258
R= .103	$\mathbf{R^2}$**=** .011				
F=1.293	**p=** .258				

The review conducted on the analysis results demonstrates that the mathematics anxiety is not a meaningful predictor of the mathematical ability competence. (R=0.103, R^2= 0.011, F=1.293, p>0.05). 1% of the total variance on the mathematical ability competence can be considered as the result of the mathematical anxiety capability. Despite their correlation, mathematical ability and mathematical achievement are different concepts. In this study, we have demonstrated the impact of the mathematical anxiety on the mathematical success. Yet, we found that ability is not relevant to mathematical anxiety. This may be resulting from the extent of the students' awareness of their mathematical abilities. Students not aware of their mathematical ability may feel anxious about mathematics or vice versa.

RESULTS AND RECOMMENDATIONS

In this study, the relation between mathematical ability and anxiety has been reviewed. The above pages contain definitions introduced by various researchers on both mathematical anxiety and ability. We have also demonstrated the negative impact of the mathematical anxiety on the students' performance at mathematic classes. Studies supporting this fact are also available in the literature. However, mathematical ability is a quite different aspect since children at this age are not capable of judging specific areas of their abilities. Consequently, in addition to other students, any student with mathematical ability will become anxious during mathematic classes unless he/she is aware of his/her ability. In order to minimize the foregoing, studies on ability should be also conducted in addition to those on the elimination of the mathematical anxiety. At this point, teachers should assume various duties and strive for drawing sufficient attention to mathematics. Similar to the students with normal intelligence level, we have determined that those with superior intelligence too may be anxious about mathematics. Their superior intelligence should not necessarily mean superior capabilities at all areas. Although mathematical ability is not described by the mathematical anxiety, the anxiety during math classes too will reduce as the required arrangements are made and the mathematical ability improves.

The following recommendations may be made according to the foregoing results;

- Areas of the students' competence should be determined and the students should be made aware.
- Seminars on mathematical ability and anxiety should be held for teachers.
- Preventive activities on mathematical anxiety should be conducted.
- Teachers should be made aware of the mathematical ability of students with superior intelligence.
- Students with superior intelligence should be thought not to fear mistakes.
- Project homework for the improvement of the students' mathematical abilities should be assigned.

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HISTORY OF MATHEMATICS FOR CREATIVE TEACHING

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Abstract

For the gifted mathematics student, creative teaching is very important. Traditional teaching methods involving demonstration and practice using closed problems with predetermined answers insufficiently prepare students in mathematics. We have a good number of successful mathematicians, but we don't use their achievements for mathematics teaching. In this study we gave an exemplary implementation. We used mathematicians' achievements for teaching mathematics and prepared a creative teaching program. The sample of study consisted of gifted students from Beyazit Ford Otosan Primary School in Istanbul. This study was put into practice at lesson and students learned better.

Key words: History of mathematics teaching, creative teaching, gifted students.

INTRODUCTION

Some People say that children are gifted only if they are in the top one percent of the population on any measure; others consider the top five percent for the identification of gifted. Obviously there is no fixed rule. The UK Department for Education and Employment is choosing to take the top five percent of schoolchildren mostly as judged by their teachers, and considers that ten percent of these underachieving. Some American states describe the top 30 percent as gifted, thought in some of the Southern US states minority children are assessed separately, taking into account their sometimes poor diet and local vocabulary. The result is that children who would actually score below average on a standardized national test are selected for special 'gifted' education in their home territory (Freeman, 2001).

The gifted curriculum must be based on science, social studies, math, and language arts, must challenge gifted learners, and must be differentiated to meet their needs. Given new standards, the teacher might consider restructuring the curriculum for the gifted learner in mathematics. Although Curriculum and Evolution Standards for School Mathematics (CSSM, 1989) was not developed specifically for gifted education, it can be used as a philosophical support to the gifted program. The National Council for Teachers of Mathematics developed the following five tenets for their standards: learning to value mathematics; becoming confident in one's ability; becoming a mathematical problem solver; learning to communicate mathematically; learning to reason mathematically (CSSM, 1989, pp. 5-6)

While these principals support mathematics education for students, they also serve as goals for education gifted learners in this content area. New mathematical knowledge emerges from problem solving that uses the student' individual abilities and allows them to reason and communicate mathematically. (Christopher, 1999). History of Mathematics would be useful for these problem solving stages. Historical analysis has been the basis for the theory that mathematics should be related to life situations. Mathematics was invented in the great civilizations in Babylonia, Egypt, Aztecs, Chinese, Indians, Greek, Romans, and Western Europe as response to social and economic needs and problems of age. In the earlier civilizations, solutions were mostly empirical, which gave rise to deductive and theoretical methods in the modern age (Clarke, 2003).

More than a century ago, Hieronymus George Zeuthen (1902) (in Furinghetti & Radford, 2002) wrote a book about the history of mathematics. Of course, this was not the first book on the topic, but what made Zeuthen's book different was that it was intended for teachers. Zeuthen proposed that the history of mathematics should be part of teachers' general education. His humanistic orientation fitted well with the work of Cajori, 1894 who, more or less by the same time, saw in the history of mathematics an inspiring source of information for teachers. Since then, mathematics educators have increasingly made use of the history of mathematics in their lesson plans, and the spectrum of its uses has widened. For instance, the history of mathematics has been used as a powerful tool to counter teachers' and students' widespread perception that mathematical truths and methods have never been disputed.

The biographies of several mathematicians have been a source of motivation for students (Furinghetti & Radford, 2002). They recommend that when teaching an appropriate topic, take a minute to tell your pupils an anecdote about one of the famous mathematicians who contributed to this particular field of mathematics. It is important for pupils to be aware of the 'human' side of these famous people. "Using biographies of mathematicians can successfully bring the human story into the mathematics class. What struggles have these people undergone to be able to study mathematics?..." (Voolich, 1993).

HISTORY OF MATHEMATICS IN EDUCATION

The area of study known as the history of mathematics is primarily an investigation into the origin of new discoveries in mathematics, to a lesser extent an investigation into the standard mathematical methods and notation of the past.

Before the modern age and the worldwide spread of knowledge, written examples of new mathematical developments have come to light only in a few locales. The most ancient mathematical texts available are Plimpton 322 (Babylonian mathematics ca. 1900 BC), the Moscow Mathematical Papyrus (Egyptian mathematics ca. 1850 BC), the Rhind Mathematical Papyrus (Egyptian mathematics ca. 1650 BC), and the Shulba Sutras (Indian mathematics ca. 800 BC). All of these texts concern the so-called Pythagorean theorem, which seems to be the most ancient and widespread mathematical development after basic arithmetic and geometry.

Egyptian and Babylonian mathematics were then further developed in Greek and Hellenistic mathematics, which is generally considered to be one of the most important in classical antiquity, for greatly expanding both the method and the subject matter of mathematics (Health, 1963). The mathematics developed in these ancient civilizations were then further developed and greatly expanded in Arabic and Islamic mathematics. Many Greek and Arabic texts on mathematics were then translated into Latin in medieval Europe and further developed there.

One striking feature of the history of ancient and medieval mathematics is that bursts of mathematical development were often followed by centuries of stagnation. Beginning in Renaissance Italy in the 16th century, new mathematical developments, interacting with new scientific discoveries, were made at an ever increasing pace, and this continues to the present day. (http://en.wikipedia.org/wiki/History_of_mathematics)

The historical development of mathematics has demonstrated that the subject is related to economic and social contexts. At every period of its development, new concepts were added, and the discipline was related to prevailing ideologies and practical needs. More than ever, today society is characterized by technological developments as its essential lifestyle. Whether it is obtaining a mortgage loan, purchasing annuities, constructing bridges, or houses, orbiting in space, communication across national boundaries on the Internet, or playing the lottery, knowledge of mathematics enables one to operate effectively (Kool, 2003).

Historical arithmetic books contain hundreds, perhaps thousands of problems. Some of these problems are suitable for use in the classroom. You can imagine that they can be solved by students, sometimes even students from primary school. But why would you ask them to do so? Why would you pay attention to the history of mathematics in your classroom? If it is just a hobby of the teacher, it can be nice and perhaps motivating but then it is comparable with the teacher who is philatelist and wants to do mathematics with stamps. Using the history of mathematics can be more than just a hobby of teacher. The history of mathematics lets children experience that mathematics is always developing, that it is continuously changing and that they are part of this evolution. Mathematics is not fixed. It didn't appear out of the blue. It is valuable for children to know about the history of mathematics, which cultures, which big names, which ordinary people have contributed to its development? What can they add to it themselves?

There are even more reasons to use the history of mathematics in classroom, even at primary school. There are many suitable problems in old textbooks. And it could be interesting to give children the opportunity to solve these problems in their own way, to construct their own solution methods. Afterwards the teacher can organize interactive classroom talks to compare and discuss the solutions of the students. In this discussion the solution method of the child that lived 450 years ago can play an interesting part. The solution of the past make children think about different solution methods, old and new ones, clever and cumbersome ones. They discover that there are many different ways to do it and they develop respect for the mathematics of their historical classmate (Kool, 2003).

In synthesizing advice from teachers, mathematics historians, mathematics educators, and preservice teachers, we identified four essential benefits. Integrating history and mathematics instruction sharpens problem-solving skills, lays a foundation for better understanding, helps students make mathematical connections, and highlights the interaction between mathematics and society. Using history lends itself naturally to connecting mathematics with other disciplines when the focus is on how mathematics has influenced world history, science, economics, inventions, and communications (Bidwell 1993; Fauvel 1991).

SUCCESSFUL MATHEMATICIANS

Every culture on earth has developed some mathematics. In some cases, this mathematics has spread from one culture to another. Now there is one predominant international mathematics, and this mathematics has quite a history. It has roots in ancient Egypt and Babylonia, and then grew rapidly in ancient Greece. Mathematics written in ancient Greek was translated into Arabic. About the same time some mathematics of India was translated into Arabic. Later some of this mathematics was translated into Latin and became the mathematics of Western

Europe. Over a period of several hundred years, it became the mathematics of the world. There are other places in the world where significant mathematics developed, such as China, southern India, and Japan. There is, of course, much mathematics being done in these and other regions (Joyce, 1998). In this study we used works of the famous mathematicians: Archimedes (287-212 B.C.), Euclid (330-275 B.C), Pythagoras (580-520 BC), Blaise Pascal (1623 - 1662), Sir Isaac Newton (1642 - 1727),Carl Gauss (1777-1855), Aryabhatta (476-550), Leonhard Euler (1707 - 1783).

CREATIVE TEACHING

During the 1960s, creativity came to misconstrued as an instant phenomenon. True enough; an apparently sudden insight may be a part of artistic and scientific discoveries. But insight is hardly sufficient by itself. It's coming to consciousness requires intense preparation, and most insight require vast testing and planning before they come to fruition (Walberg, Williams, Zeiser, 2002).

An elementary issue is: Can creativity be taught? Or are you born with it? The answer is yes and yes. Some people are born with a combination of creative genius, intelligence, extraordinary motivation, and a sense of destiny that leads them to eminent creative achievement. Mozart, Picasso, Marie Curie George Washington Carver, and Tomas Edison come to mind (Davis, 2002).

Creativity is fostered by buying low and selling high in the world of ideas – by defying the crowd. Creativity is as much a decision about and an attitude toward life as it is a matte of ability. Creativity is witnessed routinely in young children, but is hard to find in older children and adults because their creative potential has been suppressed by a society that encourages intellectual conformity. Children's natural creativity begins to be suppressed when they are expected to color within the lines in their coloring books. In essence, caregivers and teachers decide for children – and they decide to discourage natural creativity. Creative ability is what is typically thought of as creativity. It is the ability to generate novel and interesting ideas. Often, the individual considered to be creative is a particularly good synthetic thinker who makes connections between things that other people do not recognized spontaneously.

Full creativity requires a balance among analytical, creative, and practical abilities. The person who is only creative in thinking may come up with innovative ideas, but cannot recognize or sell them. The person who is only analytical may be an excellent critic of other people's ideas, but is not likely to generate creative ideas. The person who is only practical may be an excellent salesperson, but is as likely to sell ideas or products of little or no value as those ideas with genuine worth

(Stenberg, Grigorenko, 2000). At the same time, it is absolutely true that everyone's creative ability, creative productivity, and creative living can be elevated. At the very least, we all can make better use of the creative abilities with which we were born. Stenberg argued that high creativity stems from conscious decisions, for example, to redefine problems, overcome obstacles, do what you love to do – and believe in yourself.

We will organize creativity training concepts under five headings (Davis, 2002): (a) Fostering creativity consciousness and creative attitudes; (b) Improving students' understanding of creativity and creative people; (c) Exercising creative abilities; (d) Teaching creative thinking techniques (e) Involving students in creative activities

A creative attitude is at least as important as are creative thinking skills (Shank 1988, Stenberg, Grigorenko, 2000). Most teachers want to encourage creativity in their students, but they do not know how to do so. Following are twelve strategies that develop creativity. (Stenberg, Grigorenko, 2000): Redefine problems, Question and analyze assumptions, Sell creative ideas, Generate ideas, Recognize the two faces of knowledge, Identify and surmount obstacles, Take sensible risks, Tolerate ambiguity, Build self-efficacy, Uncover true interests, Delay gratification, Model creativity

In the report by Fleith (2000, Davis, 2002), classroom strategies and activities that elementary teachers believed would increase creative growth included: Cooperative groups, which expose student to differing points of view; Cluster groups, based on student interests and strengths; Allowing students to select what they want to do; Arts centers; Drawing; Giving Options; Brainstorming ; Open-ended activities; Hands-on activities, Creative writing.

Application of the creative teaching techniques referred to in this study was tried. Making use of the history of mathematics, a possible case study for creative teaching was performed for the Mathematics course.

A MATHEMATICS CURRICULUM FOR THE GIFTED AND TALENTED

Curriculum reform efforts, current research in teaching and learning, and organizational innovations to promote effective schools are vital considerations as we seek to provide educational opportunities for gifted learners (Van Tassel-Baska, 1992). A mathematics program for the gifted and talented should provide for the deserve needs of individuals and a changing society as well. With the impact of information technology, the nature of knowledge is shifting. Schools must adjust or face obsolescence. Actually, many (most) schools will not change, in my judgment, and thus our youth will be "miseducated" (Wheatley, 1983).

In designing a mathematics curriculum for the gifted and talented, many factors must be considered. Certainly the gifted and talented must have good computational skills. But computational skills must not be the beginning point. This is a fatal error that has been made in the past. Teachers have reasoned that they would introduce problem solving and advanced topics only after the computations had been mastered. This approach fails, for a variety reasons, among which is the very nature of skill learning. Skills must be practiced and maintained. A teacher can always justify devoting more time on a particular skill unless every child shows mastery. Of course this never happens. Furthermore, learning computational skills is just plain boring. In contrast, mathematics exiting and alive. By emphasizing problem solving, children can see the reason for computational skills and learn them much more efficiently. Thus computational skills need not be mastered before mathematics but should be learned simultaneously with the consideration of ideas and reasoning patterns. We just also recognize that alternatives now exist for performing complex computational topics may no longer warrant the time allocated to them. After all, "Scaring the saber-tooth tiger" continued to be taught long after the tigers had vanished (Wheatley, 1983).

Programs for the gifted often emphasize creativity or general thinking skills without attention to specific content domains. Stanley (1980, Johnson, 1994) suggested that creativity should be handled through subject areas. Learners who are gifted in mathematics require special attention within the mathematics classroom. An appropriate program for these students should adopt the following goals (Johnson, 1994):

1) To provide a context for gifted students to learn as much as possible about mathematical concepts, ideas, and skills. Since gifted students have the capacity to learn more than is usually presented in standard courses, adjustment in curriculum must be made.
2) To prepare mathematically gifted students to be creative and independent thinkers. Mathematics provides an environment for stimulating creative thought and developing the high potential of these students to become problem solvers of the highest caliber.
3) To help mathematically gifted students appreciate the beauty of mathematics. Through its study, gifted learners are more likely than other students to comprehend, value, and find meaning in mathematics as the study of patterns and the language of the universe.

Teaching mathematics has three major objectives (Gözen, 2001): (1) Teaching mathematics, (2) Demonstrating mathematical activities, (3) Causing intellectual and behavioral changes in students

For us, rather the second objective is important in terms of this study as a good instruction also aims at improving the students' awareness of mathematical activities. Such activities can be listed as follows:

- Students should be made aware of the practical benefits of this science.
- Students should be given a grasp of mathematical activity in all cases and the sciences analyzing the foregoing, along with its importance.
- Attention on the mathematical impact on fine arts should be drawn where appropriate.

An example on this subject is provided below. This activity is addressing the students' various assessment skills and was made using the mathematical history for gifted and talented students also considering different teaching techniques.

A LESSON PLAN FOR CREATIVE MATHEMATICS TEACHING

Students' attention is drawn by telling them about an upcoming "fun game". (Cards with questions quoted by famous mathematicians with their pictures affixed on the reverse side are prepared.) A ruler with zero origin is drawn on the board and other numbers which should be available on the ruler are left out. The numbers will be added once found collectively.

Below are some examples to the questions used in this study:

- The reverse side of a card with Pythagoras' picture on the front reads: "I like triangles best and I even have rule on right-angled triangles. So therefore, I'm going to ask a question about triangles. How many edges does a triangle have?

The student giving the correct answer to this question is made to count from zero on the ruler and write on the area corresponding to "three".

- The following questions are on the back side of the cards with Gauss' picture on the front: "I like to conduct space researches. Just tell me, how many planets resembling the earth are in our solar system? How many continents are on the earth? How many seas are surrounding Turkey?

Students giving the correct answers to these questions are made to mark the respective numbers on the ruler on the blackboard. The name of the respective famous mathematician in the history, asking the question, is then printed above the number.

- The question on the back of the card with Euclid's picture on the front reads: "You know, they call me the 'Father of Geometry'. Now, tell me, how many consonants are there in the phrase 'Father of Geometry'?

Many question cards are prepared based on the real lives of famous mathematicians. The ruler is completed with such cards. Active participation to the course by each student is ensured. Blanks on the ruler are completed by the students. Ideas from students on the “assignment of a name for the ruler” are obtained. Their recall of the historical mathematicians is ensured by the question as to “by whom the ruler was made.” Afterwards, the ruler is decided to be named as “the ruler of mathematicians”. The teacher writes 1cm between both numbers and explains the centimeter concept. The total ruler length is calculated upon explaining to the students the lengths between the numbers on the ruler such as 0-1 is assigned as 1cm and that between 1 and 2 as 1cm.., etc. Depending on the answers provided by the students, various measurements are made in the class using tools such as pencils, books, folders, desks, etc. Using their rulers, the students measure any object in the classroom.

The next class involves a collective drama study with the students. Two students depict Pythagoras and Pascal. Using their creativity, they make up a conversation on the invention of the calculator and ruler. After such enjoyable classes, the students will have achieved both, a good grasp of the measures of lengths and learned a lot about the lives of famous mathematicians.

RESULTS AND RECOMMENDATIONS

Mathematics is a spectrum of methods as an art like painting and music to some extent, and language to another, aiming at providing a grasp of the nature. Mathematics has a written history of 4500years. The development of mathematics within such period is divided to 5 ages during which a large number of famous mathematicians have made countless achievements. Based on the achievements, this study provides an example for creative teaching. Easily applicable during the class, this study makes the teachers aware of the importance of the mathematics history in terms creative teaching. This way, the students can enjoy and learn mathematics more effectively. Gifted and talented students have enjoyed this activity as it allows them to make an efficient use of their creativity and expressed their satisfaction of learning mathematics in a more enjoyable way. Allowing such activities in the course of the ordinary curriculum at the same time, would be also useful. Using their creativity, teachers may benefit from the mathematics history and prepare a variety of activities. Consequently, they would achieve both push the students’ creativity and make mathematics more fun. Knowledge on the mathematics history will contribute a great deal to the students’ positive approach to mathematics and result in a more solid and comprehensible course.

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TEACHING THE ETHEREAL: STUDYING TEACHING CREATIVITY IN THE MATHEMATICS CURRICULUM.

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Abstract

This report outlines a study undertaken for a Master of Arts degree from University of Auckland, New Zealand (Clack, 2006). The study proposes a framework that, when refined, could be used to describe teaching approaches. These descriptions might then be used to aid our understanding of creativity in the mathematics classroom. The study highlighted the need for curricula to be clear about what is intended by 'teaching creativity' in the mathematics classroom and how teachers' varying interpretations of creativity affect both espoused and enacted practice. The long term goal is for a refined framework to be used in teacher training, aiding teachers with identifying ways of teaching and fostering creativity effectively in the mathematics classroom.

SUMMARY OF STUDY

'Problem solving' is a prominent feature of the mathematics curriculum in New Zealand (*MiNZC*) (Ministry of Education [MoE], 1992). It is a phrase liberally used throughout the curriculum and its possible interpretations are well defined throughout. We may therefore expect 'problem solving' (in whatever guise) to be present and observable in the mathematics classroom. 'Creativity' is also emphasised as highly important in the mathematics curriculum and again, is used liberally throughout the curriculum document, so, as with problem solving, we may expect to see 'creativity' to be an observable feature of primary mathematics classes. However, while *MiNZC* suggests that creativity should be encouraged, there is little guidance to what 'creativity in mathematics' may mean or how it may be realised in the mathematics classroom. While creativity and problem-solving are given strong emphasis in the curriculum, there is a marked difference between their explanations.

A detailed study of the literature suggested that a possible link between mathematics and creativity lay in problem solving, or more specifically problem *posing*. The characteristics of problem posing outlined by Brown and Walter (2005) show many similarities with the characteristics of creativity outlined by, for example, Robinson and Koshy (2004) and Craft (2001), such as divergent problem solving and self determination. It is possible to suggest therefore that there may be an apparent link between problem solving and creativity.

If we accept the view that creativity in the classroom is ultimately dependent upon the teacher (e.g. Fisher & Williams, 2004; Robinson & Koshy, 2004) then we must explore 'teaching creativity' if we are to understand creativity in students. The apparent link between problem solving and creativity in mathematics education suggested that one approach to studying teaching creativity in mathematics may lie in using an approach to studying problem solving.

The framework for describing teaching problem solving provided by Schroeder and Lester (1989) provides concise descriptions of teaching problem solving. It outlines three teaching approaches, teaching *for* problem solving, teaching *about* problem solving and teaching *via* problem solving, each with distinct teaching characteristics. These characteristics can be displayed in table form (Appendix 1).

This framework then provided a skeleton for the construction of a framework that described teaching creativity. The framework for teaching creativity also has three teaching approaches, teaching *for* creativity, teaching *about* creativity and teaching *via* creativity also with teaching characteristics specific to each teaching approach. As with Schroeder and Lester's framework, the characteristics for teaching creativity can be demonstrated in table form (Appendix 2).

STUDY DESIGN

The small-scale study aimed to investigate problem solving and creativity in primary mathematics education using classroom observation and interview data from three Year Six (10 to 11 years olds) teachers from two schools in central Auckland, New Zealand. Data was collected through video recording three 45 minute to one hour lessons and one tape recorded interview lasting approximately 45 minutes with each teacher The interviews focused on establishing the teachers' pedagogical beliefs about problem solving and creativity in mathematics education, while video evidence was used to note typical activity in during mathematics lessons.

Data analysis used a qualitative interpretive approach to establish a set of teaching characteristics for each participant, based on their espoused and observed teaching pedagogy. In the interview, the teacher's perceptions of 'problem solving' and 'creativity' in mathematics education were discussed. This provided data for espoused teaching practices toward teaching problem solving and teaching creativity. The video data provided information for enacted teaching practices. Through observation, it was possible to note traits of teaching that could be described as characteristic of each teacher.

These characteristics were then compared to a framework for teaching problem solving as described by Schroeder and Lester (1989) (Appendix 1) and also compared to a framework for teaching creativity as proposed by Clack (2006) (Appendix 2). This provided an opportunity to evaluate the effectiveness and utility of each of the frameworks – if teaching characteristics showed close links to one particular approach, the framework could be described as a useful tool in describing teaching.

RESULTS

The match between the framework for teaching problem solving provided by Schroeder and Lester (1989) and those of the espoused and enacted data were strong, allowing each of the three teachers to be described by one of the three teaching approaches in the framework. Characteristics displayed by each teacher showed close matches to one specific teaching approach, with few outlying characteristics. Appendix 3 shows an example of how the espoused and enacted characteristics displayed by one teacher fit within one approach. This example is indicative of all three teachers. In contrast, when focusing on creativity, the match between the framework for teaching creativity and teachers' espoused and enacted practice, was less strong. Appendix 4, for example, shows how the characteristics displayed by the teacher are 'spread' across the framework and characteristics are apparent in all three of the teaching approaches.

When considering teaching problem solving, all three teachers' espoused practice showed strong matches to the enacted practice, that is, the characteristics of the teachers were present in the same teaching approach in the framework, as can be seen in Appendix 4. In contrast, there were often notable differences between interview and observation data, that is, espoused data did not match with, and even directly contradicted what was seen in the classroom. Appendix 5 provides a good example of this. 'Georgia' professed to no interest in creativity in her classroom at all, and thus none of her espoused characteristics appeared in any of the framework approaches. However, she did demonstrate many of the teaching characteristics present in the teaching *via* creativity approach. This meant that inevitably espoused and enacted characteristics could not be matched to the same teaching approach in the framework as they had in the framework for teaching problem solving.

The data indicated that the framework provided by Schroeder and Lester (1989) is a useful tool for describing approaches to teaching problem solving. The data also suggested that the framework for teaching creativity needs modifying if it is to provide closer matches and provide a useful tool for studying creativity in primary mathematics education.

DISCUSSION

The aim of constructing a framework for teaching creativity was to provide a tool by which we could describe teaching approaches in the classroom. These descriptions would help us understand what 'teaching creativity' in mathematics may mean. The data showed that the matches of teaching approaches to the framework for teaching creativity were less strong. This suggests that the framework will need modification and refining if it is to achieve its aim of providing a useful tool for describing 'teaching creativity' in mathematics education. The framework was originally constructed using information from the literature. The need for refinement may suggest that the framework needs to be built on observed practice rather than, or indeed in addition to, literature analysis.

The data often demonstrated a disparity between espoused and enacted practice for 'teaching creativity' as shown in Appendix 5, in contrast to teaching problem solving, where there was little disparity between espoused and enacted data, shown in Appendix 3. This may suggest that teaching problem solving and its implications are more widely understood than 'teaching creativity'. The disparity in espoused and enacted practice in teaching creativity could be explained by the teachers having different interpretations of creativity to one used in the study (as seen in Appendix 5) or of differing interpretations of how creativity is operationalised (Appendix 4). The multiple interpretations of creativity and the disparity between espoused and enacted practice may show the extent to which creativity in mathematics education is (perhaps poorly) understood.

The study raised three questions that need to be addressed in future research.

1) How can we be clearer about what may be intended by creativity in mathematics education?
2) How can we establish a greater common understanding and acceptance of what creativity in mathematics education is?
3) How can we establish a usable definition of creativity in the mathematics classroom?

The answer to the first two questions may merely be to encourage curriculum and policy writers to be more explicit in their interpretation and expectation of creativity in mathematics education. However, this would be over simplifying the matter. Curricula should, or indeed must be based on research evidence. It is therefore important that research into creativity is, in the future, extensive to provide evidence for such curricular content. Fisher and Williams (2004), for example, suggest that "creativity may not have an exact nature" (p. 7) and thus it may not be possible to establish an exact definition. This is not to say we cannot establish a *usable* definition of creativity for the mathematics classroom. The search for a usable definition for creativity may be far more beneficial and fruitful than an effort for a definitive, all encompassing one.

In effect, these three questions can, in fact, be treated as a single question. Without a practical, useable definition of creativity in the mathematics classroom, we are unlikely to gain a common acceptance of what creativity is and thus be clearer about what we might expect to see in the classroom. This is not to say we must work from 'top' down. Determining what we might see as 'creativity' in the classroom may lead to a common acceptance of what creativity is, and from this form a usable definition.

As a Master of Arts thesis, the research project had many limitations, most notably time and educational research experience of the researcher. Both of these will, in the near future, be addressed. The purpose of presenting this work here, therefore, is not to hold the study up not necessarily for its methodological integrity but more to demonstrate a way of studying teaching creativity in mathematics education by using an approach for studying teaching problem solving as a template for teaching creativity. This study has constructed what will hopefully, once refined, be a useful tool for future research

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Appendix 1: Characteristics of teaching problem solving

	Espoused practice	**Enacted practice**
Teaching *for*	Emphasis on knowledge, exposition of ‘facts’ or skills, with emphasis on application to real life in problem solving.	• ‘exercise’ problems and/or heavy reliance on textbooks, with ‘word problems’ prevalent • rehearsal of techniques • closed/convergent problems, or with one ‘right’ answer and method • problems use only one function • unlikely to contain material not yet learnt
Teaching *about*	Children need to be aware of how to solve problems and this is as important as domain specific knowledge when solving mathematical problems.	• discussion about how to solve the given problem(s) before attempting to find solution(s) • reference to heuristics/general problem solving skills • both convergent and divergent problems • less likely to contain new material
Teaching *via*	Children learn best through the solving of problems therefore the new material to be learnt should be presented in the context of problems.	• divergent problems • new material embedded in the problem • often more than one technique needed to solve the problem • opportunities to pose problems as well as solve problems

Appendix 2: Characteristics of teaching creativity

	Espoused practice	**Enacted practice**
Teaching *for*	Creativity occurs in the final result or the outcome of mathematical work.	• Level 1 problem posing • Mainly convergent and occasionally divergent problems • Focus on knowledge • Creativity comes from the application of knowledge in, for example, project work. • Exposition of mathematical knowledge/skills
Teaching *about*	Students need to be aware of how to be creative and be ‘creativity conscious’.	• Mainly Level 1 problem posing, occasionally Level 2 problem posing • Both convergent and divergent problems. • Discussion about what it means to be creative in mathematics and what might constitute creativity. • Explicit teaching of creativity strategies • Discussion and reference to specific strategies that may increase creativity.
Teaching *via*	The teacher provides a ‘creative role model’. Creativity is valued for its own sake.	• Level 2 problem posing prevalent • Divergent problems allowing students to pose problems. • Teacher selects problems/opportunities for ‘effective surprise’ and LCC. • Students allowed to pursue their own ideas • ‘Safe environment’ of classroom • Use of various teaching methods

Appendix 3: Teaching problem solving characteristics of Teacher one, 'John'

	Espoused practice	**Enacted practice**
Teaching *for*	Emphasis on knowledge, exposition of 'facts' or skills, with emphasis on application to real life in problem solving.	• 'exercise' problems and/or heavy use of textbooks, with 'word problems' prevalent • rehearsal of techniques • closed/convergent problems, or with one 'right' answer and method • problems use only one technique • unlikely to contain material not yet learnt
Teaching *about*	Students need to be aware of how to solve problems and this is as important as domain specific knowledge when solving mathematical problems.	• discussion about how to solve the given problem(s) before attempting to find solution(s) • reference to heuristics/general problem solving skills • both convergent and divergent problems used • less likely to contain new material
Teaching *via*	Students learn best through the solving of problems therefore the new material to be learnt should be presented in the context of problems.	• divergent problems • new/untaught material embedded in the problem • often more than one technique needed to solve the problem • opportunities to pose problems as well as solve problems

Appendix 4: Teaching creativity characteristics of Teacher two, 'Paul'

Teaching *for*	Creativity occurs in the final result or the outcome of mathematical work.	• Level 1 problem posing • Mainly convergent and occasionally divergent problems • Focus on knowledge • Creativity comes from the application of knowledge in, for example, project work • Exposition of mathematical knowledge/skills
Teaching *about*	Students need to be aware of how to be creative and be 'creativity conscious'.	• Mainly Level 1 problem posing, occasionally Level 2 problem posing • Both convergent and divergent problems • Discussion about what it means to be creative in mathematics and what might constitute creativity • Explicit teaching of creativity strategies • Discussion and reference to specific strategies that may increase creativity
Teaching *via*	The teacher provides a 'creative role model'. Creativity is valued for its own sake.	• Level 2 problem posing prevalent • Divergent problems allowing students to pose problems • Selection of problems/opportunities for 'effective surprise' and LCC • Students allowed to pursue their own ideas • 'Safe environment' of classroom • Use of various teaching methods

	Espoused practice	**Enacted practice**
Teaching *for*	Creativity occurs in the final result or the outcome of mathematical work.	• Level 1 problem posing • Mainly convergent and occasionally divergent problems • Focus on knowledge • Creativity comes from the application of knowledge in, for example, project work • Exposition of mathematical knowledge/skills
Teaching *about*	Students need to be aware of how to be creative and be ‘creativity conscious’.	• Mainly Level 1 problem posing, occasionally Level 2 problem posing • Both convergent and divergent problems • Discussion about what it means to be creative in mathematics and what might constitute creativity • Explicit teaching of creativity strategies • Discussion and reference to specific strategies that may increase creativity
Teaching *via*	The teacher provides a ‘creative role model’. Creativity is valued for its own sake.	• Level 2 problem posing prevalent • Divergent problems allowing students to pose problems • Selection of problems/opportunities for ‘effective surprise’ and LCC • Students allowed to pursue their own ideas • ‘Safe environment’ of classroom • Use of various teaching methods

BRINGING RICHER MATHEMATICAL LEARNING AND TEACHING OPPORTUNITIES USING WEB-BASED PROBLEM SOLVING ENVIRONMENTS

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INTRODUCTION

The task of the meeting learning needs of mathematically gifted students requires a deeper analysis of available resources and successful practices. The exponential growth of web-based mathematical resources in the first decade of the 21st century creates almost infinite number of educational opportunities for acceleration, enrichment, and differentiation of teaching and learning. Numerous innovative web2.0 technology tools enhances emergence of new forms of collaborative learning and teaching networks such as virtual communities, wikis, blogs, web- and pod-casting as well as video clips available via youtube and similar multimedia environments. In our paper, we will zoom in teacher and schoolchildren perceptions of educational potential of such opportunities basing on the experience with the CAMI (*Chantier d'apprentissages mathématiques interactives*) Internet project in which the 1 to 1 access to laptop computers during the whole school day allowed participation in solving rich contextual mathematical problems in 2005-2006. Interviews with participants help to get a deeper insight in the phenomenon of increasing motivation and mathematical enrichment in an interactive asynchronous weekly problem solving.

. CONTEXT AND PROBLEM STATEMENT

The francophone minority living in Atlantic Canadian provinces faces many educational issues related to the school management, curriculum development, teacher training, and work with parents. Despite a big territory and dispersed population living mainly in rural areas, the community tries to keep its cultural identity and tradition of the past while looking for a bright future for the new generations. The Internet plays an important unifying role allowing people to get access to French language resources and communicate with other members of the community. It is not by chance that the New Brunswick is the first Canadian province to install a high-speed Internet access in all its schools.

Another issue is related to the lack of educational resources in the French language due partly to the small population and connected costs of printed materials, so the province is forced to implement textbooks coming from other provinces, mainly from the Quebec. Facing so many challenges, the local educators undertook tremendous efforts to modernize a school system (QLA, 2002).

The Internet-based project CAMI has been created to fill in this gap and it had a 6 years long history of enriching mathematical problem solving practices by New Brunswick French Canadian K-12 schoolchildren before its transformation into a new virtual collaborative science and mathematics community CASMI in October 2006 (www.umoncton.ca/casmi). The development of the project is following Research Based Design methodology (Design-Based Research Collective, 2003) which includes a constant monitoring of the participants' opinions and needs using online surveys for schoolchildren who solve mathematical problems and send their solutions online and teachers who use the problems to enrich classroom practices. At the same time, we do a yearly paper and pencil survey and interviews with pre-service teachers involved into interaction with schoolchildren giving them formative feed-backs on their solutions. While data from these surveys showed relatively high level of satisfaction of all groups of participants with the quality of mathematical problems, feed-backs and Internet design as well as constantly growing number of submitted solutions (Freiman, Vézina, Gandaho, 2005, Freiman, Vézina, 2006) it remained unclear what are the reasons of such optimistic opinion, what learning occurs and how it is enriched by the project related activities. In our proposal, we limit us to the analysis of perception of 20 participants, 4 teachers and 16 schoolchildren about the problems, Internet design and interaction, feed-back, and knowledge building and sharing process.

The New Brunswick Laptop Initiative provided almost 200 Grade 7 and 8 schoolchildren from 6 provincial schools (3 of them were French schools) with individual laptops and everyday wireless access to the Internet during the school day in 2005-2006. Our research data registered a CAMI site among the resources the most frequently used in mathematics (Fournier et al., 2005). Therefore, we assumed that interviewing in depth teachers and students about their perception of mathematical problems and related online interaction would let to learn more about mathematical enrichment opportunities created by the CAMI project.

THEORETICAL AND METHODOLOGICAL CONSIDERATIONS

While a number of educational systems are moving toward inclusive classrooms like for example in New Brunswick, Canada (Mackey, 2006), more variety of resources are to be used in order to meet particular needs of diverse learners. ICT (Informational Communicational Technology) can play an important role in providing differentiation in the inclusive classroom. Kenewell (2004) lists several approaches making ICT use valuable in supporting special learning needs namely differentiation by task (like giving different roles to students working on the same project), by response (allowing able students to go beyond the basic learning objectives making and testing their own conjectures), by support (adjusting to the level of student's understanding), and by resource (allowing preparation of a range of resources each of them can be produced and adapted meeting particular learning needs).

Rotigel, Fello (2004) advance that 'readily accessible classroom computers, supervised access to the Internet, and appropriate software programs offer opportunities for gifted students to advance at their own rate'. A cognitive potential of such well known ICT-based tools as word-processing, spreadsheets, web-based search engine is well documented, recent studies reveal new opportunities of collaborative and communicative aspects of relatively new virtual environments such as blogues, discussion forums, wikis, video-, web- and pod-casting (Depover, Karsenti, Komis (2007). What kind of learning and teaching opportunities for mathematically able students and their teachers and parents can they provide?

One type of opportunities is closely related to the concept of the online collaboration which helps to accomplish several outcomes according to Palloff, Pratt (2005): assessing with deeper levels of knowledge generation, promoting initiative, creativity, and critical thinking, allowing students to create a shared goal for learning and forming the foundation of a learning community, addressing all learning styles, and helping creating more culturally sensitive classroom. The combination of all mentioned factors may lead to a deeper, more efficient, and complete learning process so is potentially beneficial to fostering mathematical giftedness in its largest sense.

While the work of assessing the educational impact of these innovative approaches is about to begin, there is already an important body of research evidence of several successful patterns. Nason, Woodruf (2004) studied computer supported collaborative learning (CSCL) environments based on model-eliciting problems that provide a rich context for mathematical knowledge-building discourse that in its turn enables students to adequately represent mathematical problems and to facilitate student-student and teacher-student hypermedia-mediated discourse. Among other, virtual tools help students to generate diverse solutions and solution processes to the same mathematical problem and communicate both synchronously and asynchronously diverse solutions to others.

Math Forum is another example of rich online mathematical resources spread out into over a million and a half pages with variety of services including *Problem of the Week*, *Ask Dr. Math* and *Teacher2Teacher* sections (Renninger, Shumar, 2004). The research about success of this community points that besides this huge amount of resources, a particular culture of learning is being created collaboratively by all participants: schoolchildren, students, pre-service teachers, classroom teachers, and math and pedagogy experts. In fact, all participants are not only passive users of resources but rather active contributors to the co-construction of new community mathematical knowledge. Moreover, results from several studies conducted by the authors and other researchers suggest that interactivity and communication about mathematical problems are the key advantages technology can bring in fostering making connections of participants to serious mathematics content, engaging them into questioning and finding solutions and providing them with models for working with challenging problems and topics (Renninger, Shumar, 2004). According to recent studies of mathematical giftedness such environments can bring more enriching and challenging opportunities to mathematically gifted learners in and beyond the classroom (Sheffield, 2004).

Mathematical enrichment by means of virtual learning communities has been studied within the framework of the NRICH project whose main impact on the pupils was in terms of '*helping them to gain a wider appreciation of mathematics and raising the profile of mathematics as a subject that could be interesting enough to pursue either within or outside school or for further study*' (Jones, Simons, 2000). The structure of communication and discussion available within virtual learning communities enables young people to look deeper into more complex and philosophical questions like for example *Agora de Pythagore* project in which middle school students were puzzled with several geometric construction problems asking what could lead mathematicians of Antiquity to explore them in depth. Web-based discussion between pupils led them to several explanations like simplicity of the straight line and the circle, role of symmetric configurations constructed with compass and ruler, as well as simplicity and shared acceptance of these two basic tools by all members of mathematical and non mathematical community. Pallascio (2003) argues that all these discoveries were possible because of the learning context which is thus favourable to the development of higher order thinking abilities.

Results of cited above research suggest that virtual environments may be suitable for enrichment and differentiation in the mixed-ability classroom in which mathematically gifted students can be provided with necessary challenge. Following are some theoretical foundations of the CAMI project. In our work we are inspired by Brousseau's theory of didactic situations (1998) and Sierpinska's theory of mathematical understanding which stresses that to access to the superior level of her understanding of mathematics, the student needs to reorganize his knowledge. But we can not just tell the student how to reorganize her knowledge but we can engage her in new situation-problem, preparing ourselves for any kind of difficulties and obstacles and helping the student to overcome them (Sierpinska, 1994).

In her model of development mathematically promising children, Sheffield (1999) opted for use of multi-dimensional tasks within the scope of heuristic and open model which would contribute to the development of abilities to create, to make links, to investigate, to communicate, and to evaluate. Briefly saying, in order to succeed in teaching mathematics, it is necessary to create a good equilibrium between the routine and well structured tasks and those more creative and innovative. The students need to create a solid repertoire of positive experiences in problem solving, which would allow them to develop their self-confidence and potential (Klein, 2003).

At the same time, while some children are struggling with mathematics, others might loose their interest in mathematics because they do not have enough challenge (Tempest, 1974). At the same time, the research shows that all children when they are young are eager to learn more and are able to learn more. Our study of conducted in one Montreal – based private elementary school, proved to be another case showing that an appropriate challenge can not only foster a development of mathematical potential in gifted students in early grades but it could also be beneficial for all students (Freiman, 2003, 2004).

Many authors demonstrate a need for more challenging curriculum not only for mathematically talented but for a larger population of schoolchildren. Sheffield (1999) calls these children 'mathematically promising' and argues that a challenging program may also introduce them to the joys and frustrations of thinking deeply about a wide range of original, open-ended, or complex problems that encourage them to respond creatively in ways that are original, fluent, flexible, and elegant (Sheffield, 1999).

Within our CAMI project, we use the Problem of the Week model used in a similar way to the MathForum site (*mathforum.org*). Four weekly problems according to four levels of difficulty are being posted on our web site. School children have a week to solve some of these problems and to submit their solutions electronically. A personal e-feed-back is provided by pre-service teachers after that. The most interesting solutions and the names of successful problem solvers are shared with other project members via a common e-space. Teachers can also opt to integrate problems in their own way in and beyond the classroom. Our research findings from 2004-2006 student-teacher (pre- and in-service) surveys reveal the importance of mathematical communication turns out to be an important discovery pre-service teachers make while working with the CAMI within mathematics education courses. This evidence is confirmed by schoolchildren who say that this feed-back helps them to improve their problem solving skills. Our findings also showed that the university students find CAMI beneficial for the development of mathematical communication in young children. This communication is fruitful when it is bidirectional. That means that pupils are encourage communicating their mathematical ideas using a variety of tools. Pre-service teachers should be able to understand this variety, to appreciate it and to guide children through their problem solving process (Freiman, Vézina, 2006).

In our presentation we will triangulate these results with data collected during semi-structured interviews with teachers and students who participated actively in the project during 2005-2006 school years. The interviews were conducted in 3 French schools. Participants were all 4 participating teachers and 16 students selected on the volunteer basis by teachers, 2 students (one boy and one girl) from each of 4 Grade 7 classes and 4 Grade 8 classes. All interviews were semi-structured and asked the same set of questions to all participants. In 30-minutes interviews, teachers were asked about their perception of the use of CAMI as teaching and learning resource, students' motivation to solve problems, quality of problems and feed-backs, quality of the web-design, observation of students' learning, obstacles and challenges, suggestions for the improvements. In 15-minute interviews, schoolchildren were asked what they do like in the CAMI projects, how interesting they find the problems, what they do think about solutions and feed-backs and how they compare this activity to traditional forms of classroom mathematics. They were also asked about the web-design and suggestions of modifications. According to the teachers, all students were successful in mathematics and needed more enriched opportunities. All interviews were audio-recorded and word transcription has been made to allow thematic qualitative analysis. The themes rose by teachers and students were compared to allow triangulation of data.

BRIEF OVERVIEW OF COLLECTED DATA

One of the students mentioned that she likes problems because they make her think. In fact, they contain a challenge an sometimes are difficult to solve which is good according to this student: '*problem has to be a problem*'. Another student adds to this '*sometimes problems are easy to solve, sometimes they are very difficult, and you have to think a lot, in order to solve them*'. Her colleague remembered a problem about animals that took her 4-5 days to solve. But afterward, she felt very proud of herself of being the only one from her class to succeed. Seeing the name published on the Internet in case of successful solution seems to be motivating according to some students. One of the teachers said that '*there is no need to motivate students to solve problems*' which '*have good variety of mathematical content and are complex enough to challenge the students*'. In the same order of ideas, one student argued that solving problems online is much more motivating than having them from the textbooks of 70-s.

According to one teacher, students surprise her sometimes with ingenious solutions and strategies: '*I was pleased getting with one student who was telling me a very original way to solve a problem*'. Sometimes, the problem required facts that have not been learnt in class. In this case, both, teachers and students point at the

Internet as source of missing information and they often like to share it with their peers: '*One of my Grade 7 students has to use a Pythagorean theorem that we didn't study before, she found it on the web and presented it to the rest of the group*'. On teacher shared an observation made on one student who found that CAMI editing tools limited her in the presentation of her solution, so she made the problem in her blog and put the e-link to it in the CAMI solution form.

According to the teachers, CAMI project is a good source of differentiation. One teacher mentioned that having four difficulty levels is good because students could choose an appropriate problem. When students do not succeed they can try another one at the easier level. At the same time, one teacher cited a student who needed more challenge, so this student went to see the most difficult problem and succeeded in solving it correctly. Overall, teachers agree that '*Each student can move at her own pace through the problem solving processes'*.

Regarding feed-backs received from the university students, both teachers and students agreed on their utility and depth. One teacher said that comments her students receive are more detailed than ones she usually provides for them. Her colleague sees some magic in this fact that someone else looks at student's solution and comments it. One of interviewed students argued in the same way: '*It is excellent that someone analyses our solutions and gives us a feed-back*', her colleague adding: 'The comments I get help me to become a better problem solver'. Teachers said that students are very proud when they get a positive comment and like to share it with them: It is like they have won a cup in a competition.

Sometimes problems lead to discussion between students in the classroom: '*Students like discussing CAMI problems and different ways of solving them'*. On student mentioned that sometimes she can get help from other peers to get some boost in the problem solving process.

DISCUSSION AND CONCLUSION

Our brief description of collected data suggests several themes emerging from the interviews that need to be analysed and discussed in the connection to our research question which was to get a deeper understanding of how participants of the online mathematical problem solving project CAMI perceive teaching and learning opportunities provided by the web-based technology. This insight, will nurture further discussion on what could these opportunities allow in term of fostering mathematically gifted students in a mixed-abilities classroom.

First theme that emerges from the interviews is an overall excitement of both teachers and students about CAMI as source of rich and challenging mathematical problems provided by technology. While this same fact is neither new nor surprising (teachers and students were already actively engaged in the project), their comments let us attribute it to several factors: good variety of problems and contexts going beyond the minimum fixed in the school curriculum, possibility of choice of the problem according to student's learning needs and personal interest, nice and attractive (and interactive) environment comparatively to traditional types of resources, a possibility of getting a formative feed-back from pre-service teachers (not from YOUR teacher). In addition to this, some socio-affective factors emerge as well: intrinsic motivation, proud of success, risk-free environment with no assessment related sanctions and stress, possibility of sharing discoveries with peers and teachers. It is also to mention that 1 to 1 access to the laptops and wireless Internet in the classroom facilitated the harmonious use of the resource which seems to be another important factor contributed to positive response.

Another issue that comes from our findings is related to the problem of differentiation in the mathematics classroom. As we said in our previous sections, today's mathematics curriculum recognizes differences in how children learn and the right of each child to receive an education that is adapted to her needs. The New Brunswick math curriculum stresses that in order to meet educational need of each student; teachers have to use a variety of approaches. CAMI presents one possible resource that gives each pupil a chance to choose an appropriate problem, solve it at her own pace using her own strategy and communication tool. It brings also some informal elements in the classroom routine. The fact that each participant gets a personal comment from the university student can be seen as motivating factor for schoolchildren because they see that their work brings attention of other people and is being socially valorized by a personal attention or even a public recognition (children can see this recognition when their solution is posted as interesting or their name is placed on the congratulation list). However, more research is needed in order to study in more details the factors of intrinsic motivation among participants of the CAMI project.

Our results seem to concord with other research in the field that attributes to the technology the role of the valuable learning tool as well as the catalyst of inspiration and independent learning. While this role gives benefits to all students, it helps the gifted once to reach the appropriate depth and breath of curriculum and advanced product opportunity (Johnson, 2000). The fact that the CAMI virtual environment allows bringing mathematical learning experiences beyond the classroom and regular curriculum is also beneficial for gifted students who require tasks and learning environments that differ substantively from the regular mathematical programs (Diezmann, Watters, 2002).

A particular interest of students to work in virtual environments can also be attributed to the particular characteristics of today's generation of learners, which is called the Net Generation Oblinger, Oblinger (2005). The needs of mathematically gifted students coming from this generation have not been studied.

But the latest development of the CAMI project toward the virtual collaborative learning communities incorporates several new virtual tools of web 2.0 technology which includes more integrated multimedia representational tools (like inserting pictures, links to other multimedia resourced including videos, tools for expression emotions like 'smilies') along with community membership with shared and personalized e-access using logins and personal cyber-portfolio as well as tools of knowledge sharing and collaboration (micro-communities and discussion forum) may bring additional options for mathematically gifted learners (Freiman, Lirette-Pitre, Manuel, 2007) that requires deeper research investigations.

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MATHEMATICS IN AN ARTS/TECHNOLOGY-RICH SETTING

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Abstract

In this paper we report on a study of a mathematics program for grade 7-8 gifted students that integrated rich mathematics tasks with the arts (poetry, music and drama) and technology. The program was offered partially online and partially in a classroom setting.

INTRODUCTION

Gifted students are characterized by their ability to give sustained attention to problem solving, by their propensity to question, experiment and explore, by their inventive solution strategies, and by their ability to engage their imaginations to generate highly original ideas. However, gifted students spend most of their time in regular classrooms with their same-age peers (Westberg, Archambault, Dobyns, & Salvin, 1993) working on mathematics activities that fail to challenge them (Howley, Pendarvis & Gholson, 2005). Although gifted students need and tend to seek challenging tasks (Stanley, 1991) regular classroom mathematics activities tend to leave them bored (Feldhusen & Kroll, 1991; Galbraith, 1985; Gentry, Gable & Springer, 2000; Gallagher, Harrandine & Coleman, 1997; House, 1987). This lack of mathematical challenge also leaves gifted students with a shallow and naïve understanding of mathematics (Gentry, Gable & Springer, 2000).

In this paper we report on a study of a mathematics program for grades 7-8 gifted students that encouraged the use of imagination (Egan 1997) by integrating rich mathematics tasks with the arts (poetry, music and drama) and technology. The program was offered partially online and partially in a classroom setting. The study sought to investigate (1) students' perceptions of the program, and (2) students' mathematical thinking patterns.

Setting

The study involved 32 grades 7 and 8 gifted students from a rural school district. The students were normally integrated in regular classrooms with their same-age peers, with 2-4 gifted students per class. They were occasionally withdrawn from the regular classroom and offered enrichment activities. Sometimes they were brought together as a group for project-based activities.

The context for the study was a short mathematics program, where students came together as a group for an initial full day of mathematics activities, followed by several weeks of participating in an online discussion forum, and then a final full day of mathematics activities. The mathematical activities were open-ended and were designed to offer students opportunities to make mathematics connections and experience the complexity of mathematical ideas. The activities integrated poetry, music, drama and technology. For example, the first activity was introduced using the song Flatland (available at http://www.edu.uwo.ca/mathscene/T/flatland.html), then students explored lines and triangles on a spherical surface by drawing on balloons, and then they wrote their own poems (sample shown below) and performed them as songs. The sphere helps students experience new and wonderful perspectives of ordinary concepts. Straightness becomes complex, and although there are no extrinsic straight paths on a sphere, there do exist intrinsic straight paths (geodesics or great circles) (Henderson, 2001). The concept of parallel also increases in complexity in the context of spherical geometry. There are no parallel lines on a sphere from an extrinsic perspective, but there are lines that "feel" parallel, and are intrinsically parallel, such as lines of longitude, which we often depict as straight, parallel lines on flat maps of the Earth. And if we form a "triangle" using three geodesics on a sphere, the sum of its angles is not 180^{o} but varies within a range of values.

Students had access to a computer lab and they wrote their poems in the online discussion forum so that they may be accessed between sessions by their peers. The online discussion forum allowed for Wiki (peer-editatble) postings, offering students opportunities for collaborative authoring and knowledge construction. The discussion forum also allowed for rich text, student drawings using a built-in drawing tool, and the embedding of images.

At the end of each activity, students recorded what they learned and what they felt during the activity. In the final day, students also created and performed skits to depict some of the things they learned and felt. Between sessions, the online forum was used to continue working on the activities of the first session. As well, new activities were introduced for online exploration.

THEORETICAL FRAMEWORK

The study adopts a sociocultural perspective based on Vygotsky's (1978) view that knowledge is constructed in interactions with others. By "others" we also refer to digital tools that permeate our new media culture. We see humans-with-media as actors in the production of knowledge (Borba & Villareal, 2005).

METHODOLOGY AND METHODS

The study used a constructivist grounded theory framework (Charmaz, 2000) because of its emphasis on the cyclical nature of gathering the data and then using the data to generate insights, hypotheses and further questions, which fits well with the shape and purpose of this research. Grounded theory methodology employs techniques of induction and deduction to develop theory.

Data was collected from the following sources: (1) observation and field notes; (2) videotaping of key activities in the face-to-face sessions; (3) students' learned/felt summaries at the end of each activity; (4) postings in the online discussion forum; and (5) focus group interviews/discussions. A content analysis was conducted to identify themes based on the questions of the study and to identify representative excerpts. These initial themes identified are presented below. In the next stage of analysis, the data will be read a second time to validate the themes.

Parallel, parallel, parallel ☺

opposites attract,

but parallels repel.

We expected hands on

We expected more work

But we found ourselves drawing on balloons

With teachers gone berserk

One sang us a song ♪

With a base guitar ♫

we proved the theory wrong ✗

we felt powerful rarrr! ✲

INITIAL FINDINGS

Students' perceptions of the regular mathematics classroom

The regular mathematics classroom

The gifted students in our study described everyday mathematics learning experiences that supported the research findings summarized at the beginning of this paper. They reported that they spent most of their time in regular mathematics classrooms, where "(t)eachers over-explain ideas and tend to go on and on when teaching mathematics." They described math class as a process of taking up homework, going over a new concept in too much detail/repetition, and then having homework time in which they were expected to help others (thus not leaving enough time for them to do their own homework). The mathematics classroom role of many of the gifted students in our study was that of "extra teachers in the classroom, and we are expected to help other students with seatwork." Most students found this role difficult because other students either constantly asked them for help or they rejected their assistance.

Favourite subjects

When we asked the students in our study to tell us which school subjects they enjoyed the most, they typically did not select mathematics. Many students identified language arts and drama as their favourite subjects. "Mathematics is about right and wrong answers" and "not as free as drama and language arts." Language arts and drama also tend to be "project-based" which they found to be "more appealing." Some students selected physical education, because "people get what to do" and therefore the whole class can move to playing games and scrimmaging more quickly. In math class, "we have to wait a long time before we actually do anything."

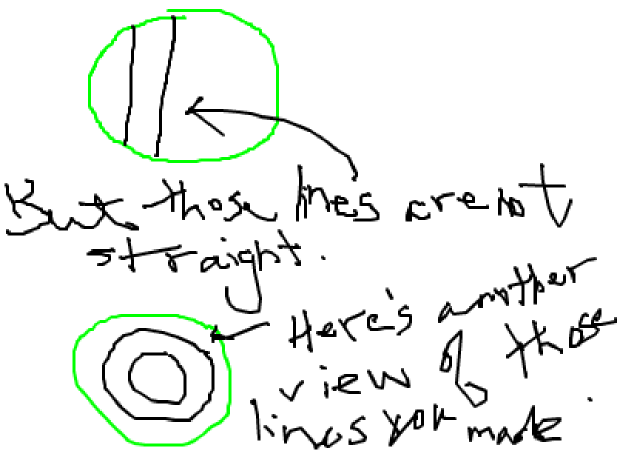
Students' perceptions of the math/arts/technology program

Mathematical challenge.

Many students expressed that the activities were challenging. "I felt challenged today because I really needed to use my imagination and personal theories to answer some of the questions." Some students also initially felt overwhelmed by the new ideas. "Today I felt a bit overwhelmed at first. But once I understood, I felt great to be learning so much." "I felt confused at first but then I started to get it and had lots of fun." Generally, students "felt more comfortable with this program" and appreciated the greater challenge of " really getting to the bottom of interesting and new concepts," of "really trying to understand."

Mathematical surprise.

Students expressed surprise to learn that mathematics concepts can be much more complex than they initially imagined. They were surprised "about how much there is to learn about parallel lines on a sphere and on a flat surface" and that "there might be more than one answer to a mathematical question." When we asked the gifted students to elaborate on mathematical concepts prior to the activities, like what it means for two lines to be parallel, there exhibited a lack of depth of understanding, with a focus on facts or definitions, rather than mathematical relationships and connections.



Learning atmosphere

In the second face-to-face session, we provide students with a choice of activities and gave them the responsibility of deciding when their investigations were complete. Students expressed that they felt "respected and empowered" due to having to choose the activities and the degree of completion for each activity selected. "You could really use your imagination." There was also a sense of relief on their part that they were not expected to do a given number of questions from a particular page in the text book like they do normally in mathematics class. Students stated a variety of other reasons for liking the program:" we could state our opinions and thoughts freely"; "I was learning and having fun at the same time!" ; "The art twist was cool!!"; the hands on nature of the activities helped them to "think through ideas"; and, the online discussion allowed them to "use other people's ideas to create your own new ones". It is interesting to note that some reported that when parallel lines were discussed in their classrooms, they felt as though they knew something that no one else knew. However, they didn't speak up and challenge the "truth" that parallel lines never meet.

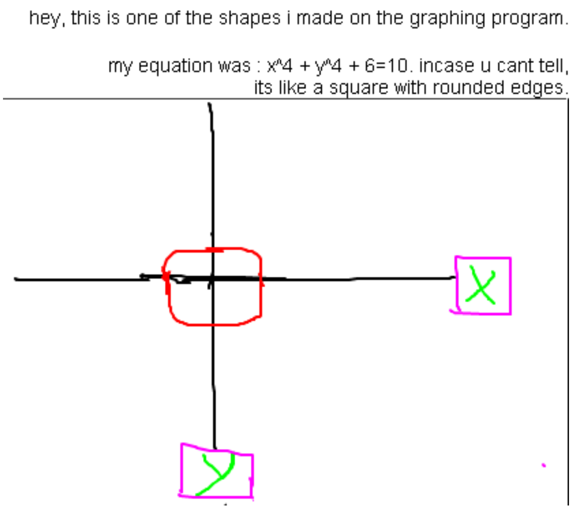
Students' mathematical thinking

Using drawings to illustrate ideas

Students used the drawing tool in the online discussion forum to communicate their thinking to others. For example, the diagrams on the right depict two perspectives of lines of longitude.

Using interactive tools

The students had available a variety of online interactive tools for exploring math concepts. For example, the diagram on the right depicts the graph of $x^4+y^4+6=10$ that a student discovered while using an online plotting program.



New ways of talking about math

It is interesting that as we circulated from group to group, we heard students using some of the phrases that were part of the songs that we sang together. For example, some students started using the phrases "this makes my mind click" and "this makes my mind fly", which were part of the lyrics of one of the songs.

Affective uses of technology

Some students used the online discussion board to stay in touch with one another between sessions and to develop virtual mathematical relationships. Some of the students used the drawing tool to offer their peers virtual gifts, such as the strawberry shown on the right. Some drew self-portraits and shared them with the group (example shown on the left). They also made regular use of emoticons to express their feelings.

DISCUSSION

One solution to programming for the gifted is to provide challenging mathematical experiences that involve them in thinking deeply and creatively about mathematical relationships. This would involve structuring at least some experiences around projects or open-ended tasks rather than content topics, cutting across mathematical strands and other subject areas, and offering some choice of investigations to be pursued. What we are describing is perhaps a solution for programming for all students of mathematics. In fact, although our focus in this paper is on gifted students, the mathematics program that forms the basis for our study is one that we have also offered in regular mathematics classes, as well as in mathematics courses for teachers, and mathematics courses for parents. Given that a majority of gifted students spend most of their time in regular mathematics classrooms with their same-age peers, one way of providing a richer mathematics experience is to offer a richer mathematics experience form all students.

Acknowledgement

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